# AN EFFECTIVE DIMENSIONAL INSPECTION METHOD BASED ON ZONE FITTING

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## KEYWORDS

Dimensional inspection, geometric tolerance, zone fitting

## ABSTRACT

In geometric tolerancing, minimum zone evaluation technique is used when the measured data is regarded as a copy of actual surface and the tolerance zone is represented as geometric constraints. Since functional requirements or assembly conditions on a manufactured part are normally translated into geometric constraints to which the part must confirm, it is appropriate to use zone fitting technique to evaluate part quality. This paper presents an effective dimensional inspection method based on zone fitting. With rigid body transformation, computational geometry, and nonlinear optimization techniques, the tolerances of geometric entities can be determined. The developed methodology can be applied to evaluate 2D and 3D geometry. It can be used for dimensional inspection with datum.

## INTRODUCTION

Coordinate measuring machines (CMM) are widely used to generate data points from manufactured part surfaces. The generated measurement data must be analyzed to yield critical geometric deviations of the part according to the requirements specified by the designer. Since ANSI/ASME standards do not specify the methods that should be used to evaluate dimensional tolerances, different verification algorithms may be used and different results can be obtained. For example, the min-max fit generally returns a smaller maximum deviation than the least squares fit. Thus, given a set of CMM measurement data, the part could be accepted based on min-max fit method but rejected based on least square fit method (Choi and Kurfess, 1999).

Functional requirements or assembly conditions on a manufactured part are normally translated into geometric constraints to which the part must confirm. A tolerance zone is a region of space constructed by offsetting (expanding or shrinking) the object's nominal boundaries (Requicha, 1983). Tolerance conformance is achieved when the measured points fit into the tolerance zone. Although the tolerance zone representation is not fully compatible with the ANSI/ASME standards, it is more intuitive and can be easily associated with CAD models.

Many researchers have developed zone-fitting algorithms based on different techniques. The preferred approach adopted by most of them is to model the zone-fitting problem as a nonlinear optimization problem. An objective function is defined; and a solution that optimizes the objective function is sought. Murthy and Abdin (1980) proposed several methods such as Monte Carlo technique, normal least squares fit, simplex search, and spiral search techniques to evaluate the minimum zone deviation. Depending on the requirement and the problem, the individual technique or a combination of the above techniques could be applied to achieve the desired accuracy. Shunmugam (1987) proposed an approach for evaluating form errors (minimum zone) of engineering surfaces based on the minimum average deviation (MAD) principle. A stray peak or valley on the actual feature introduces considerable variations in the results obtained by the minimum deviation method.

Carr and Ferreira (1995a, 1995b) developed algorithms which solve a sequence of linear programs that converge to the solution of the nonlinear optimization problem. Different linear programs were used for flatness, straightness, and cylinericity verification models. Two flatness verification methods were proposed. One method searches for the reference plane so that the maximum distance of each data point from this plane is minimized. Another method searches for two parallel supporting planes so that all data points are below one plane, above the other, and the planes are as close as possible. Tsukada and Kanada (1985) used direct search methods such as the improved simplex method and Powell's method for minimum zone evaluation of the cylindricity deviation. Kanada and Suzuki (1993) use the downhill simplex method and the repetitive bracketing method to evaluate the minimum zone flatness considering the respective convergence criteria. Huang et al. (1993) developed a new minimum zone method for evaluating straightness and flatness errors based on the control line rotation scheme and the control plane rotation scheme. The search for the best fit plane or line starts with the least squares plane or line as the initial condition.

Another general approach to computing the minimum zone solution is based on the computational geometry theory. Lai and Wang (1988) evaluated the straightness and roundness based on the convex polygon methods. Hong and Fan (1986) proposed an eigen-polygon method. Etesami and Qiao (1990) use the two-dimensional (2-D) convex hull of the data points to solve the 2-D straightness tolerance problem. Swanson et al. (1994) proposed an optimal algorithm to evaluate the out-of-roundness factor, which determines the extent to which a planar shape deviates from a circle. This algorithm also makes use of the medial axis and farthest neighbor Voronoi diagram, but does not require their intersection, thereby yielding the improvement in complexity. Roy and Zhang (1992) proposed a mathematical formulation determining the roundness error, which exploited the properties of convex hull and Voronoi diagrams to produce a faster algorithm for establishing the concentric circles. Traband et al. (1989) compute the three-dimensional (3-D) convex hull and searches for the minimum zone of the convex hull to compute flatness. These methods are guaranteed to find the minimum zone solution but at a great computational cost (Carr and Ferreira, 1995).

Most of the algorithms discussed above do not use the equation of the nominal surface specified by the designer in their evaluation of the minimum zone value. The algorithms evaluate the form tolerance. The location and the nominal size of the feature is not a concern. Fitting of measured points to nominal geometries has been studied by many researchers. Patrikalakis and Bardis (1991) and Gosh (1992) developed data localization algorithms to NURBS and sculptured surfaces. Choi and Kurfess (1999a) introduced a zone fitting algorithm that can be used to verify the conformance of measured data to a given set of tolerance zones. The zone fitting algorithm is further extended for minimum zone evaluation (Choi and Kurfess, 1999b). The work by Chio and Kurfess demonstrated that the tolerance zone fitting method is a powerful tool for go/no-go conformance decision-making. In addition, determining the minimum zone provides the required information for manufacturing process control.

Based on the data localization approach (Choi and Kurfess, 1999), this paper presents an effective tolerance zone fitting method using computational geometry algorithms and nonlinear optimization techniques. The work is unique in terms of using the same methodology to fit a specific geometric future and to fit a complete part model. The method also addresses the issue of part inspection with the consideration of datum. In the following sections, the method for tolerance zone fitting is first presented. Tolerance zone fitting examples for 2D and 3D geometric features are shown. Application of the present method for turbine blade inspection with specified datum is also demonstrated.

## DIMENSIONAL INSPECTION USING TOLERANCE ZONE FITTING

In the present study, the zone fitting algorithm uses the equation of nominal surface and evaluates if the measured data fit into the boundary of a given zone. Since the coordinates of the CMM data is in a different reference from that of the ideal surface, the data undergo a rigid body transformation to be placed in the same reference frame as that of the design model. The process is known as data localization and is shown in Figure 1.

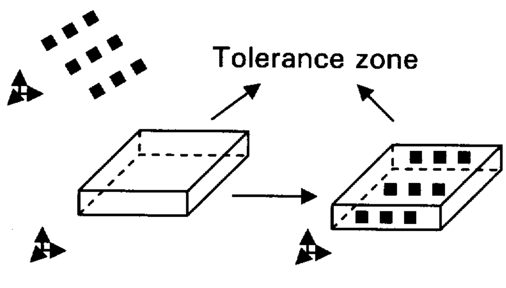



Figure 1. Data Localization

The data localization process can be achieved by homogeneous transformation (Choi and Kurfess, 1999). Consider a surface S(x,y,z) in the 3-dimensional space. Let P be the set of points measured from the part surface. To place P in the same reference plane of the design model, P is transformed to P* by a matrix H.

$$P = \{p_i, i = 1 \dots n\} \text{ n : number of measured points}$$

$$H = \begin{bmatrix} c\theta c\phi & c\theta s\phi & -s\theta & t_x \\ -c\psi s\phi + s\theta s\psi c\phi & c\phi c\psi + s\theta s\psi s\phi & s\psi c\theta & t_y \\ s\psi s\phi + s\theta c\psi c\phi & -s\psi c\phi + s\theta c\psi s\phi & c\theta c\psi & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

where $s = \sin$, $c = \cos$, $\psi, \theta, \phi$ are rotation angles about the X, Y, and Z axes respectively, and $t_x, t_y, t_z$ are the translations along X, Y, and Z axes respectively. Thus:

$$P^*(\theta,\phi,\psi,t_x,t_y,t_z) = \{u_i \in R^3 | u_i = H(R,t) \times p_i\}, i = 1 \dots n \quad (2)$$

where $R^3$ represents the 3-D space.

Individual tolerance zone ($f_i$) is created around the individual surface ($s_i$) based on the bilateral tolerances specified for that surface. Let the inner and outer tolerances be $d_{in}$ and $d_{out}$. Therefore, we have

$$T(S, d_{in}, d_{out}) = \{u \in R^3 | -d_{in} \le dist(u,S) \le d_{out}\} \quad (3)$$

where $dist(u,S)$ is the distance from point $u$ to surface $S$. The tolerance zone T is a union of all the individual tolerance zones ($f_i$) while S is the union of the individual surfaces ($s_i$). For a given S, T and P, H is to be determined such that P* lies in T i.e. P* belongs to T(S, $d_{in}$, $d_{out}$). To find the transformation that places the measured point set P in to the tolerance zone T constitutes a nonlinear optimization problem. The function is defined as:

$$N(u, T(S, d_{in}, d_{out})) = \begin{cases} 0 & \text{if } u \in T(S, d_{in}, d_{out}) \\ 1 & \text{if } u \notin T(S, d_{in}, d_{out}) \end{cases} \quad (4)$$

If a point lies in the tolerance zone the value of the function is 1, and if the point is outside the tolerance zone the value of the function is 0. Therefore, the function N (u, T) is a Boolean function. Since we have to find the transformation that places the points in the tolerance zone T, the function N (u, T) can be used as the objective function. Since the minimum state of the function N gives the feasible domain, optimization can find the rigid body transformation that places the points in tolerance zone T. The optimization problem is modeled as follows:

$$Min\ Obj = \sum_{i=1}^{n} N(u_i, T(S, d_{in}, d_{out})) \quad (5)$$

The solution of optimization gives the transformation parameters $(\theta, \phi, \psi, t_x, t_y, t_z)$ that minimize the objective function. The objective value becomes zero when all the points lie in the zone and has a non-zero value if any one of the points is outside the zone.

## Modified Point Location Method

The point location method (Preparata and Shamos, 1988) serves the purpose very well if the problem is to check whether a point lies in the convex polygon. In the new zone fitting algorithm presented in this paper, geometric features such as circle, composite 2-D geometries which are a collection of higher order curves, and 3-D planes are tackled. Therefore, the point location method (PLM) is modified to satisfy the requirements of the new algorithm. Determining if a point is within a given boundary, that is finding the value of Eq. (4), both 2-D and 3-D cases are demonstrated.

**Modified PLM for 2-D Geometries.** As an example, consider a tolerance zone defined by two circles as shown in Fig. 2. The concentric circles are presented as follows:

$$(x-a)^2 + (y-b)^2 = r_i^2$$
$$(x-a)^2 + (y-b)^2 = r_o^2 \quad (6)$$

where $(a,b)$ is the center and $r_i, r_o$ are the inner and outer radii respectively.

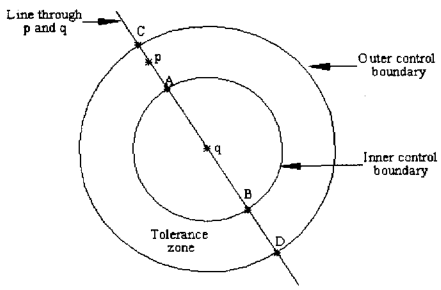



Figure 2. Modified PLM for 2-D Geometries

To determine if a point p lies in the tolerance zone, the equation of the line passing through points p and q, any point inside the inner control boundary, is:

$$a_1x + b_1y + c_1 = 0 \qquad (7)$$

Solving equations (6) and (7) simultaneously, the points where the line and the control boundaries intersect are found. The line passing through *p, q, A,* and *C* can be represented in the parametric form:

$$x = (1-t)x_1 + tx_2$$
$$y = (1-t)y_1 + ty_2 \qquad (8)$$

In the current case, the value of parameter *t* is zero at point *q* and *t* is one at point *p* i.e. the starting point of the line is *q* and the end point is *p*. Using Eq.(8) the values of parameter *t* at points *A* and *C* are calculated. The point *p* lies in the tolerance zone if and only if the following conditions are satisfied: $t_A \le 1, t_C \ge 1$, where, $t_A$ and $t_C$ are values of *t* at points *A* and *C* respectively. The same approach is followed while dealing with composite 2-D geometries which are a collection of 2-D curves. The above procedure is employed for each of the curve in the collection.

**Modified PLM for 3-D Geometries.** Similar to the procedure followed for the 2-D geometry, modified PLM is demonstrated to find out whether the point lies between two planes. Consider two parallel planes represented as follows:

$$a_1x + b_1y + c_1z + d_1 = 0$$
$$a_2x + b_2y + c_2z + d_2 = 0 \qquad (9)$$

where $a_1 = a_2, b_1 = b_2, c_1 = c_2$

These two planes form the inner and outer control boundaries forming the tolerance zone as shown in Fig. 3. We have to find whether the point *p* lies in the tolerance zone. To determine that the equation of the line passing through points *p* and *q* is found out. The equation of the line in 3-D is of the form:

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1} \qquad (10)$$

To find the points where the line and the control boundaries intersect, (9) and (10) are solved simultaneously. Let the line intersect the outer plane at *A* and the inner plane at *B*. Now the line passing through *q, p, A* and *B* is represented in the parametric form. The parametric form is given by:

$$x = (1-t)x_1 + tx_2$$
$$y = (1-t)y_1 + ty_2$$
$$z = (1-t)z_1 + tz_2 \qquad (11)$$

In the current case, the value of parameter *t* is zero at point *q* and *t* is one at point *p* i.e. the starting point of the line is *q* and the end point is *p*. Using (11) the values of parameter *t* at points *A* and *B* are calculated. The point *p* lies in the tolerance zone if and only if the following conditions are satisfied: $t_A \ge 1, t_B \le 0$, where $t_A$ and $t_B$ are values of *t* at points *A* and *B* respectively. Using the same approach, point location problem for 3-D parametric surfaces can be solved.

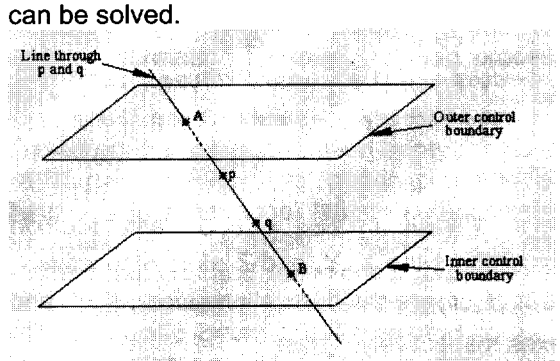



Figure 3. Modified PLM for 3D geometries

**Tolerance Zone Fitting Algorithm**

To find the minimum zone values for the geometric features under dimensional inspection, the algorithm has to solve the nonlinear optimization problem repeatedly. The inputs to the algorithm are:

1. Measured set of points from the part surface (P). (3.5)
2. The equation of the nominal geometric feature (S)
3. The inner and the outer (bi-lateral) tolerance limits ($d_{in}$, $d_{out}$). The control boundaries i.e. the tolerance zone (T) will be constructed by expanding and shrinking the nominal boundaries of the feature.

4. Initial estimate ($h_{init}$) of the six transformation parameters that place the points in the zone.
5. Tolerance for convergence ($\varepsilon$) of the zone fitting process.

Fig. 4 outlines the framework for the new zone-fitting algorithm in the form of a flow chart. To search for the feasible domain in the six-dimensional parameter space, the objective function in the nonlinear unconstrained optimization problem is minimized. The objective function either returns a zero (all points fit in the zone) or a non-zero value (any one point does not fit into the zone). In the case that the objective function returns zero, the tolerance can be further shrunk. Note that $d_{in\_L}$ and $d_{in\_U}$ are the lower and upper limits of the inner boundary and $d_{out\_L}$ and $d_{out\_U}$ are the lower and upper boundary of the outer boundary in the binary search. Similarly, if the objective function reports a non-zero value, out of tolerance can be concluded. The zone can also be further expanded (doubled) to find the minimum zone. The optimization is implemented using MATLAB. The BFGS Quasi-Newton method with a mixed quadratic and cubic line search produce was used in the MATLAB function.

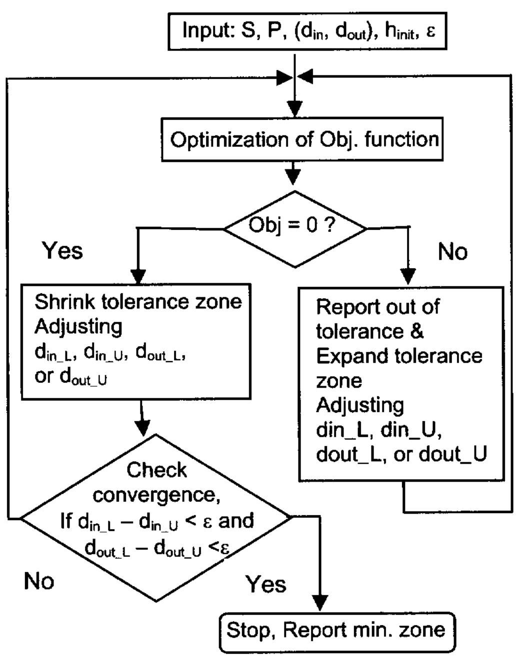



Figure 4. Tolerance Zone Fitting Flow Chart

**Dimensional Inspection with Datum**

Due to the complex geometries of certain 3-D objects such as a turbine blade, carrying out tolerance assessment becomes a difficult task. To inspect such parts, often a collection of 2-D cross-sections with respect to a datum (reference plane) is inspected. The bilateral tolerance specifications for the 3-D geometry are applied to each 2-D cross-section. Thus the tolerance zones are constructed for each cross-section by offsetting the nominal 2-D geometry of the particular cross-section, with the bilateral tolerance values. The tolerance zone is effectively represented in *2½ D*.

In the proposed zone-fitting algorithm, the rigid body transformation that places the points in the tolerance zone can be constrained. In other words, the degrees of freedom enjoyed by the set of points are reduced. In the turbine blade example, assuming the stacking axis is Z-axis, the set of points is allowed to translate along the X and Y axes and rotate about the Z-axis only. Thus by constraining the transformation, the zone-fitting algorithm determines whether the points measured at the 2-D cross-sections fit in the respective tolerance zones simultaneously. However, to address any uncertainty in the height measurement, the set of points may be allowed to translate along the z-axis within a certain range. The constrained transformation methodology is demonstrated using the turbine blade as an example in the next section.

## DIMENSION INSPECTION EXAMPLES

This section presents the dimension inspection using the proposed zone fitting method. The results are compared to that of least square method and the zone fitting method by Choi and Kurfess (modified for bi-lateral tolerance).

**The 2-D Line Inspection**

Fig. 5 shows the nominal 2-D line and the tolerance zone constructed by offsetting (expanding and shrinking) the nominal boundary (line) based on the bilateral tolerance limits. The offset values are: $d_{in}$ = -1.5 and $d_{out}$ = 1. The data is collected by simulating the line model in CAD software and generating 21 points. The rigid body transformations are determined by three methods – the zone-fitting method proposed by Choi and Kurfess (1999), the least squares fit and the new zone-fitting algorithm proposed in the present study. The rotational transformation parameters are in radians while the translational parameters are non-dimensional. Since the line model is defined in 2-D space, not all transformation parameters are evaluated. Only, translation along the X and Y axes and the rotation about the Z-axis is permitted. Table 1 gives the transformation variables, which place the set of points in the tolerance zone, determined by the three different methods.

Figure 5. Inspection of a 2-D Line

Table 1 Transformation Variable for 2-D Line

| Trans. variables | Choi and Kurfess | Least Sq. Fit | Pres. Meth. |
|---|---|---|---|
| $\phi$ (radians) | -0.1958 | 0.034598 | 0.000 |
| $t_x$ | -0.000372 | 0.000680 | 0.000 |
| $t_y$ | 0.28727 | 0.055640 | 0.000 |

The residual deviations of the zone-fitting method proposed by the new zone-fitting method and the least squares fit are shown in Figs. 6 and 7. The residual is the distance of a point from the nominal or the fitted geometry. The dotted lines represent the corresponding tolerance zone boundaries while the solid lines represent the minimum zone. The decision whether the measured set of points satisfy the tolerance specifications is made based on the results yielded by the different fitting methods. The zone-fitting method proposed by Choi and Kurfess (1999) and the new zone-fitting algorithm fit the points in the tolerance zone but the least squares fit gives a decision that the 2-D line is out of tolerance.

The conflicting results help us conclude that the tolerance conformance definition must govern the selection of the verification algorithm. If it is to be determined whether the points fit in the tolerance zone, zone-fitting algorithm must be employed.

## The 3-D Cylinder Inspection

In the case of the 3-D cylinder (Fig. 8), three nominal geometries are specified – the cylinder (zone 1), the top plane, and the bottom plane (zone 2, 3). The tolerance zones are individually constructed based on the respective bilateral tolerance specifications. For every point it is determined whether it lies in any of the tolerance zones. In this way the minimum zone values for each tolerance zone are determined.

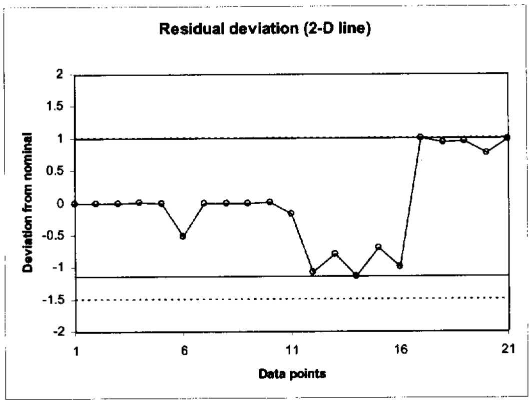



Figure 6. Residual in 2-D Line Inspection (present method)

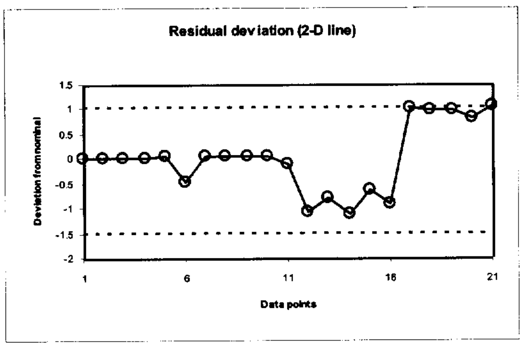



Figure 7 Residual in 2-D Line Inspection (Least Squares Fitting)

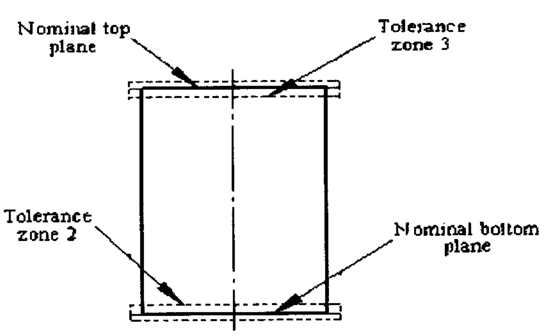
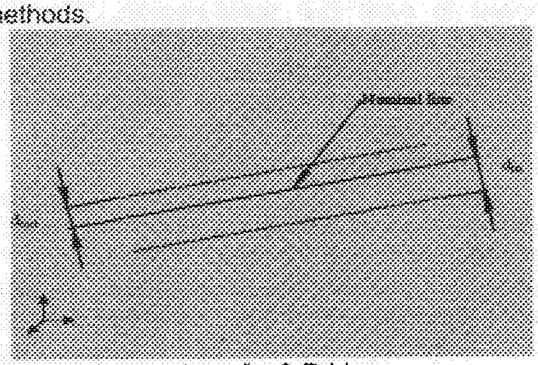



Figure 8. Inspection of a 3-D Cylinder

The data is collected by simulating the 3-D cylinder model in CAD software and generating 44 points for the 3-D cylinder (circular surface, the top and bottom planes). The rigid body transformations are determined by Choi and Kurfess (1999) and the present method. Table 2 evaluated. Only, translation along the X and Y axes and the rotation about the Z-axis is permitted. Table 1 gives the transformation variables, which place the set of points in the tolerance zone, determined by the three different methods.

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The conflicting results help us conclude that the tolerance conformance definition must govern the selection of the verification algorithm. If it is to be determined whether the points fit in the tolerance zone, zone-fitting algorithm must be employed.

**The 3-D Cylinder Inspection**

In the case of the 3-D cylinder (Fig. 8), three nominal geometries are specified – the cylinder (zone 1), the top plane, and the bottom plane (zone 2, 3). The tolerance zones are individually constructed based on the respective bilateral tolerance specifications. For every point it is determined whether it lies in any of the tolerance zones. In this way the minimum zone values for each tolerance zone are determined.

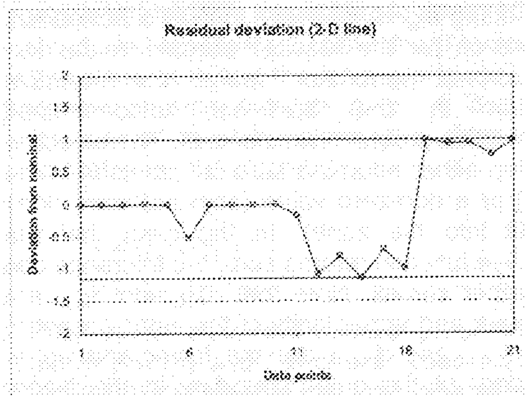


Figure 6. Residual in 2-D Line Inspection (present method)

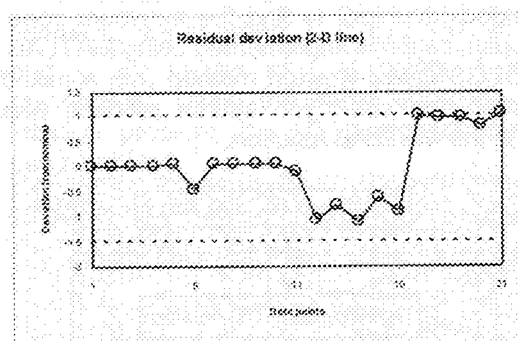


Figure 7 Residual in 2-D Line Inspection (Least Squares Fitting)

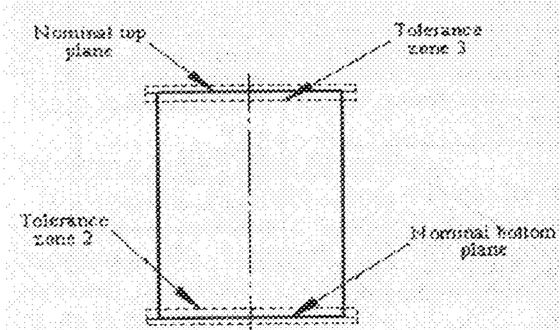


Figure 8. Inspection of a 3-D Cylinder

The data is collected by simulating the 3-D cylinder model in CAD software and generating 44 points for the 3-D cylinder (circular surface, the top and bottom planes). The rigid body transformations are determined by Choi and Kurfess (1999) and the present method. Table 2 gives the transformation variables, which place the set of points in the tolerance zone.

Table 2. Transformation Variable for 3D Cylinder

| Transformation variables | Choi and Kurfess | Pres. Meth. |
|---|---|---|
| $\psi$ (radians) | 0.0002374 | 0.0277 |
| $\theta$ (radians) | 0.0007541 | 0 |
| $\phi$ (radians) | -0.003137 | -0.0020 |
| $t_x$ | 0.077277 | -0.0671 |
| $t_y$ | 0.017196 | 0.0323 |
| $t_z$ | -0.079719 | -0.0342 |

The zone-fitting method proposed by Choi and Kurfess and the new zone-fitting algorithm fit the points in the tolerance zone, but the minimum zone values evaluated are different. Table 3 gives the minimum zone values calculated by the two methods. The values given by the new zone-fitting algorithm are higher than that of the zone-fitting method proposed by Choi and Kurfess. The difference can be attributed to the different objective functions. The method proposed by Choi and Kurfess relies on specifying a convergence tolerance, while the new zone-fitting algorithm uses computational geometry technique (modified PLM) and a Boolean function for zone fitting. The ambiguity whether the point lies inside or outside the tolerance zone is completely eliminated.

Table 3. Minimum Zone Values for 3D Cylinder

| | Choi and Kurfess | | | Present Method | | |
|---|---|---|---|---|---|---|
| | Cylin. | Top | Btm | Cylin. | Top | Btm |
| Min $d_{in}$ | -1.066 | -1.566 | -1.707 | -1.708 | -1.718 | -1.654 |
| Min $d_{out}$ | 1.191 | 1.582 | 1.629 | 1.152 | 1.651 | 1.718 |
| Zone | 2.258 | 3.148 | 3.336 | 2.860 | 3.370 | 3.372 |

## The Turbine Blade Inspection

In the present study, the turbine blade geometry model adopted is that proposed by Pritchard (1985). He proposes that to uniquely define an airfoil cascade on a cylinder requires only eleven parameters and the immediate result is a nozzle or rotor with analytically defined surfaces. These eleven independent parameters are found to be necessary and sufficient for creating an airfoil. These parameters translate into five points and five slopes on the cylinder of given radius. The suction and the pressure surfaces are best described by third order polynomials. This model can successfully create airfoils at different heights from the hub of the blade. By stacking these airfoil sections about the stacking axis, the 3-D model of the turbine blade is developed (Figure 9). Table 4 gives the mathematical representation for each piece of the airfoil section. Suffices *LE*, *TE* and *C* denote leading edge, trailing edge and circular arc respectively.

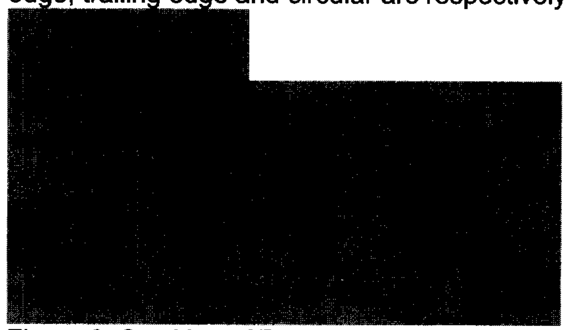

Figure 9. Stacking of Turbine Cross sections

Table 4 Equations Representing Airfoil Sections

| | Mathematical function |
|---|---|
| Leading edge circle | $(x-a_{LE})^2+(y-b_{LE})^2=r_{LE}^{\;2}$ |
| Trailing edge circle | $(x-a_{TE})^2+(y-b_{TE})^2=r_{TE}^{\;2}$ |
| Circular arc | $(x-a_C)^2+(y-b_C)^2=r_C^{\;2}$ |
| Suction surface | $y=a_1x^3+a_2x^2+a_3x+a_4$ |
| Pressure surface | $y=b_1x^3+b_2x^2+b_3x+b_4$ |

The data is collected by simulating the 3-D turbine blade model in CAD and generating 20 points for each airfoil section. A total of six sections, at different heights (with respect to the datum), determine the complete 3-D turbine blade. The bilateral tolerance values specified are $d_{in}$ = -0.0002 and $d_{out}$ = 0.0002. The rigid body transformations parameters are determined by the new zone-fitting algorithm proposed in the present study. The rotational transformation parameters are in radians while the translational parameters are non-dimensional. Since the circle model is defined in 2-D space, not all transformation parameters are evaluated. Translation along the X and Y axes and the rotation about the Z-axis is permitted. The minimum zone and the transformation that places the points in the respective zones simultaneously are given in Tables 5.

Table 5 Min. Zone and Values of Transformation variables for Turbine Blade

| | | | |
|---|---|---|---|
| Min din | -0.00015547 | $\phi$ (rad) | 4e-06 |
| Min dout | 0.00015547 | $t_x$ | 0 |
| Min Zone | 0.00031094 | $t_y$ | -2.34e-07 |

gives the transformation variables, which place the set of points in the tolerance zone.

Table 2. Transformation Variable for 3D Cylinder

| Transformation variables | Choi and Kurfess | Pres. Meth. |
|---|---|---|
| $\psi$ (radians) | 0.0002374 | 0.0277 |
| $\theta$ (radians) | 0.0007541 | 0 |
| $\phi$ (radians) | -0.003137 | -0.0020 |
| $t_x$ | 0.077277 | -0.0671 |
| $t_y$ | 0.017196 | 0.0323 |
| $t_z$ | -0.079719 | -0.0342 |

The zone-fitting method proposed by Choi and Kurfess and the new zone-fitting algorithm fit the points in the tolerance zone, but the minimum zone values evaluated are different. Table 3 gives the minimum zone values calculated by the two methods. The values given by the new zone-fitting algorithm are higher than that of the zone-fitting method proposed by Choi and Kurfess. The difference can be attributed to the different objective functions. The method proposed by Choi and Kurfess relies on specifying a convergence tolerance, while the new zone-fitting algorithm uses computational geometry technique (modified PLM) and a Boolean function for zone fitting. The ambiguity whether the point lies inside or outside the tolerance zone is completely eliminated.

Table 3. Minimum Zone Values for 3D Cylinder

| | Choi and Kurfess | | | Present Method | | |
|---|---|---|---|---|---|---|
| | Cylin. | Top | Btm | Cylin. | Top | Btm |
| Min $d_{in}$ | -1.066 | -1.566 | -1.707 | -1.708 | -1.718 | -1.654 |
| Min $d_{out}$ | 1.191 | 1.582 | 1.629 | 1.152 | 1.651 | 1.718 |
| Zone | 2.258 | 3.148 | 3.336 | 2.860 | 3.370 | 3.372 |

## The Turbine Blade Inspection

In the present study, the turbine blade geometry model adopted is that proposed by Pritchard (1985). He proposes that to uniquely define an airfoil cascade on a cylinder requires only eleven parameters and the immediate result is a nozzle or rotor with analytically defined surfaces. These eleven independent parameters are found to be necessary and sufficient for creating an airfoil. These parameters translate into five points and five slopes on the cylinder of given radius. The suction and the pressure surfaces are best described by third order polynomials. This model can successfully create airfoils at different heights from the hub of the blade. By stacking these airfoil sections about the stacking axis, the 3-D model of the turbine blade is developed (Figure 9). Table 4 gives the mathematical representation for each piece of the airfoil section. Suffices *LE*, *TE* and *C* denote leading edge, trailing edge and circular arc respectively.

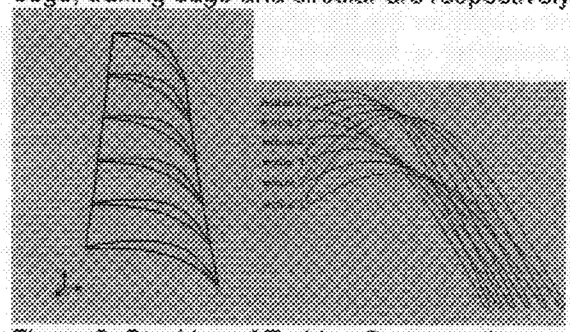

Figure 9. Stacking of Turbine Cross sections

Table 4 Equations Representing Airfoil Sections

| | Mathematical function |
|---|---|
| Leading edge circle | $(x-a_{LE})^2+(y-b_{LE})^2=r_{LE}^2$ |
| Trailing edge circle | $(x-a_{TE})^2+(y-b_{TE})^2=r_{TE}^2$ |
| Circular arc | $(x-a_C)^2+(y-b_C)^2=r_C^2$ |
| Suction surface | $y=a_1x^3+a_2x^2+a_3x+a_4$ |
| Pressure surface | $y=b_1x^3+b_2x^2+b_3x+b_4$ |

The data is collected by simulating the 3-D turbine blade model in CAD and generating 20 points for each airfoil section. A total of six sections, at different heights (with respect to the datum), determine the complete 3-D turbine blade. The bilateral tolerance values specified are $d_{in}$ = -0.0002 and $d_{out}$ = 0.0002. The rigid body transformations parameters are determined by the new zone-fitting algorithm proposed in the present study. The rotational transformation parameters are in radians while the translational parameters are non-dimensional. Since the circle model is defined in 2-D space, not all transformation parameters are evaluated. Translation along the X and Y axes and the rotation about the Z-axis is permitted. The minimum zone and the transformation that places the points in the respective zones simultaneously are given in Tables 5.

Table 5 Min. Zone and Values of Transformation variables for Turbine Blade

| | | | |
|---|---|---|---|
| Min din | -0.00015547 | $\phi$ (rad) | 4e-06 |
| Min dout | 0.00015547 | $t_x$ | 0 |
| Min Zone | 0.00031094 | $t_y$ | -2.34e-07 |

## CONCLUSIONS

A new zone-fitting method was developed to evaluate the tolerances of geometric features. It determines the rigid body transformation that places the set of points measured from the actual surface in the specified tolerance zone. The search for the transformation parameters is modeled as a nonlinear optimization problem. The objective function is defined in such a manner that no convergence tolerance needs to be set. Thus it eliminates the ambiguity whether a point is in or out of the tolerance zone. Given the nominal surfaces, the developed algorithm evaluates if the measured points lie in the specified tolerance limits, and further determines the minimum zone in which the measured set of points lie. This provides vital information as to the actual part quality. Based on the assessment, the manufacturing process can be adjusted. The developed algorithm is employed to evaluate the form tolerance of a 2-D line, 3-D cylinder and a turbine blade. By constraining the transformation parameters, the proposed methodology can be used for dimension inspection with datum.

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