

# Process-oriented tolerancing for multi-station assembly systems

YU DING,<sup>1</sup> JIONGHUA JIN,<sup>2</sup> DARIUSZ CEGLAREK<sup>3</sup> and JIANJUN SHI<sup>4</sup>

<sup>1</sup>*Department of Industrial Engineering, Texas A&M University, College Station, TX 77843, USA*  
E-mail: yuding@iemail.tamu.edu

<sup>2</sup>*Department of Systems and Industrial Engineering, The University of Arizona, Tucson, AZ 85721, USA*  
E-mail: jhjin@sie.arizona.edu

<sup>3</sup>*Department of Industrial Engineering, University of Wisconsin—Madison, Madison, WI 53706, USA*  
E-mail: darek@engr.wisc.edu

<sup>4</sup>*Department of Industrial and Operations Engineering, The University of Michigan, Ann Arbor, MI 48109, USA*  
E-mail: shihang@umich.edu

---

In multi-station manufacturing systems, the quality of final products is significantly affected by both product design as well as process variables. Historically, however, tolerance research has primarily focused on allocating tolerances based on the product design characteristics of each component. Currently, there are no analytical approaches to optimally allocate tolerances to integrate product and process variables in *multi-station manufacturing processes* at minimum costs. The concept of process-oriented tolerancing expands the current tolerancing practices, which bound errors related to product variables, to explicitly include process variables. The resulting methodology extends the concept of “part interchangeability” into “process interchangeability,” which is critical due to increasing requirements related to the selection of suppliers and benchmarking. The proposed methodology is based on the development and integration of three models: (i) the tolerance-variation relation; (ii) variation propagation; and (iii) process degradation. The tolerance-variation model is based on a pin-hole fixture mechanism in multi-station assembly processes. The variation propagation model utilizes a state space representation but uses a station index instead of a time index. Dynamic process effects such as tool wear are also incorporated into the framework of process-oriented tolerancing, which provides the capability to design tolerances for the whole life-cycle of a production system. The tolerances of process variables are optimally allocated through solving a nonlinear constrained optimization problem. An industry case study is used to illustrate the proposed approach.

## 1. Introduction

Manufacturing operations are inherently imperfect in fabricating parts and assembling products. Product imperfections were first described in the framework of part interchangeability which was introduced and implemented in early mass production systems. This then led to the development of product tolerancing. Tolerancing is a primary means to guarantee part interchangeability. There is a significant body of literature related to tolerancing methods and their applications. Summaries of the state-of-the-art, the most recent developments, and the future trends in tolerancing research can be found in Bjorke (1989) and Zhang (1997) as well as in a number of survey papers, for example, Chase and Parkinson (1991), Roy *et al.* (1991), Jeang (1994), Ngoi and Ong (1998), and Voelcker (1998).

In general, product errors accumulate over the whole manufacturing process and can be divided into two major stages (Fig. 1): (i) part fabrication processes, such as

the stamping process (forming processes); the machining process (material removal processes), or rapid prototyping (material deposition processes) that transform raw materials into components or parts; and (ii) the assembly process that joins all the parts into the final product.

Traditionally, tolerance analysis and synthesis have been studied in both stages in the context of product variables, i.e., the focus is on part interchangeability. We feel that there are tremendous needs to further expand the technique to the interchangeability of manufacturing processes. This is becoming increasingly apparent with increasing requirements related to manufacturing best practices, supplier selection and benchmarking (where each supplier may use a different process to manufacture the same product) or outsourcing. Tolerancing has the potential to be an important tool in such developments. We propose to extend the scope of tolerancing to explicitly include process variables in manufacturing processes.

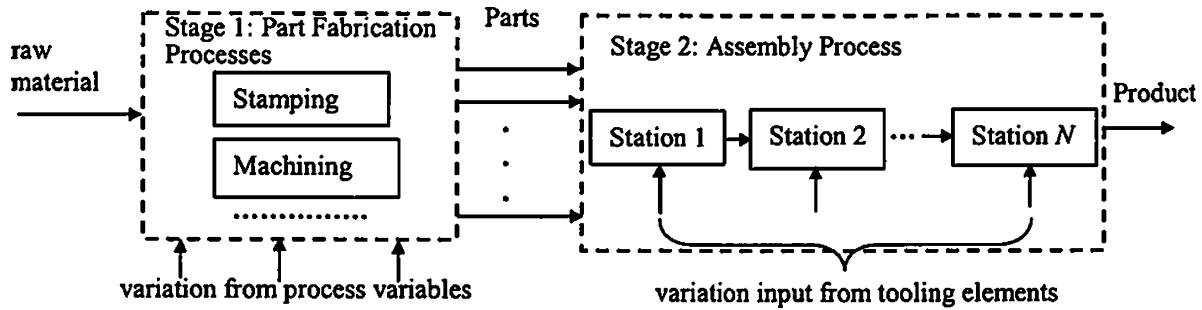


Fig. 1. Schematic diagram of a manufacturing process.

### 1.1. The two stages of traditional tolerancing

Tolerance synthesis is usually conducted separately in the part fabrication and product assembly processes (Zhang, 1996). Tolerancing at stage-2 is often called *tolerancing-for-assembly*, i.e., the tolerance requirements of a finished product are allocated to the dimensions of individual parts. The allocated part tolerances are called *design tolerances*. According to Voelcker (1998), tolerancing for assembly “has been the predominant concern in most product designs for at least half a century.”

Tolerancing at stage-1 involves converting design tolerances into *manufacturing tolerances*, i.e., the tolerances of intermediate working dimensions in part fabrication processes, such as in machining processes. The main methodologies used for tolerancing in stage-1 are based on tolerance charting (Ngoi and Ong, 1993, 1999). As an illustration, let us consider the following example in Fig. 2(a and b). The product design dimensions ( $D1, D2$ ) with their tolerances ( $T1, T2$ ) are shown in Fig. 2(a). The manufacturing process involves two operations to remove materials and generate the required dimensions (Fig. 2(b)). Accordingly, there are two working dimensions ( $WD1, WD2$ ) that are the direct result of these two manufacturing operations. The purpose of tolerance allocation at stage-1 (using tolerance charting) is to establish the relation between  $D1, D2$  and  $WD1, WD2$  and to transform design tolerances  $T1, T2$  to manufacturing tolerances  $WT1, WT2$ .

### 1.2. Product variables and process variables

*Product variables* are those key variables that characterize a design so that it satisfies specified product functional requirements. Product variables are also called Key Product Characteristics (KPCs). They include the design dimensions of finished assemblies/parts as well as the working dimensions of intermediate workpieces. The dimensions on a product blueprint are considered to be product variables because they are the output of manufacturing actions as opposed to direct descriptions of the process status. A closer examination reveals that working dimensions ( $WD1, WD2$ ) and their tolerances ( $WT1, WT2$ ) are product variables.

On the other hand, *process variables* are not part of the product information. *Process variables* are those key variables that characterize the process that controls specified product design variables. Process variables are also called key control characteristics. They describe the working condition of the tools that are used to hold or fabricate a workpiece during machining or assembly processes.

Let us consider another example as shown in Fig. 3 to illustrate the meaning of a process variable. This example is similar to the process studied in Rong and Bai (1996) and Choudhuri and DeMeter (1999). In this example, the important question is how the variation caused by locator tolerance  $PT1$  affects the quality of a machined part. The dimension and tolerance of the locator are indicators of the working condition of a fixture element rather than the

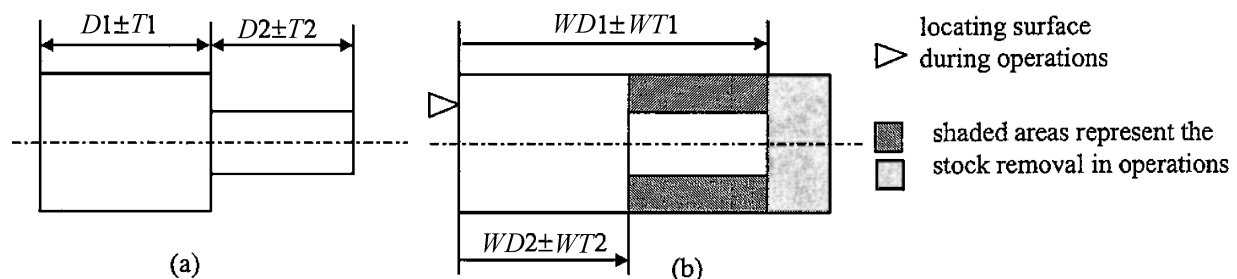


Fig. 2. (a) Design tolerance; and (b) manufacturing tolerance.

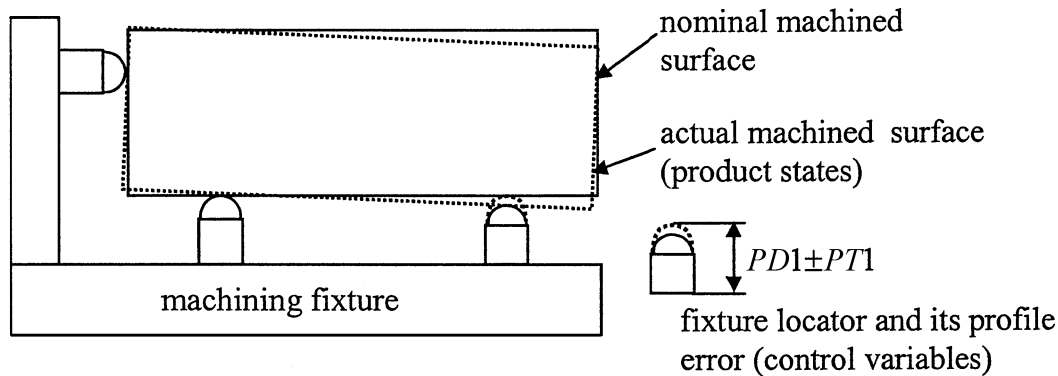


Fig. 3. The effect of the process variable (locator error) on product quality.

descriptions of the machined part. Thus, the dimension and tolerance of the locator ( $PD1$ ,  $PT1$ ) are process variables.

The difference between the product variables ( $WD1$ ,  $WD2$ ) and the process variable ( $PD1$ ) can be explained as such: the tolerances of process variables describe the *cause* of manufacturing imperfections whereas the tolerances of product variables describe the *effect* of variations in the process variables on product dimension or other quality characteristics. In more complex processes with multiple stations and/or operations, the tolerance of product variables is also affected by variations in the process variables of earlier operations/stations. For example, when the locating surface (indicated by a triangle) in Fig. 2(b) is subject to profile errors, the working tolerances  $WT1$  and  $WT2$ , which use the indicated surface as the machining datum, will be affected by the propagation of variation from previous operations that generate the locating surface.

The process variables that were discussed in the above examples are dimensional or geometrical variables. However, process variables can generally include a broad category of physical variables associated with manufacturing processes (in both part fabrication and assembly processes).

### 1.3. Product-oriented tolerancing vs process-oriented tolerancing

Although it is a common knowledge that tolerancing controls process imperfections and inaccuracies, the process information that is included in the tolerancing-for-assembly at stage-2 is actually limited to some heuristic information presented implicitly in the form of cost-tolerance functions, e.g., information about machine availability or capability in a combined process (Fig. 7.2 in Bjorke (1989)). Thus, the tolerancing-for-assembly can be described as "product-oriented" tolerancing.

Process information such as manufacturing sequence and tool condition is seemingly included in tolerance charting at stage-1. However, based on the discussions in the previous section, tolerance charting is still product-oriented since the intermediate working dimensions ( $WD1$ ,  $WD2$ ) and their tolerances are product variables. In other words, process

information is actually included in product-oriented tolerancing in indirect or implicit ways.

In this paper, instead of only considering product variables, we propose to explicitly include process variables, such as the locator dimension and tolerance in Fig. 3, in the tolerancing scheme. To differentiate this approach from traditional tolerancing, we call it *process-oriented tolerancing*. The tolerancing techniques studied in Rong and Bai (1996) and Choudhuri and DeMeter (1999) can be considered as to be process-oriented techniques.

It is a mainstream perception that process variables are not major variation contributors in assembly processes. Assembly research often assumes that variations originate from individual components and the tools used in the assembly processes only function as an auxiliary mechanism to roughly hold and position parts before they are assembled. One may quickly conclude that process-oriented tolerancing is not an applicable concept to assembly processes. This conclusion is in fact not true. In general, assembly processes can be classified into two types (also refer to Mantripraganda and Whitney (1999) for this classification): (i) type-I assembly, where parts are assembled through part-to-part mating surfaces, which is consistent with the aforementioned perception; and (ii) type-II assembly, where parts are positioned by fixtures while there is no part-to-part interference to prevent a part from being freely positioned by the fixture. Figure 4(a and b) shows examples of both type-I and type-II assembly processes. A significant amount of the research in the *tolerancing-for-assembly* has considered type-I assembly processes whereas only limited research has been conducted on tolerancing for type-II assembly processes. However, there is a large class of type-II assembly processes that include automotive or aircraft body assembly and printed circuit board assembly processes.

The final dimensional accuracy of type-II assemblies are determined during the assembly process whereas the accuracy of type-I assemblies is mainly determined by the precedent fabrication processes of each part. In contrast to type-I assembly, the product quality of type-II assemblies is significantly affected by variations in the tooling elements, especially in the case of fixture locators (Celgarek and Shi,

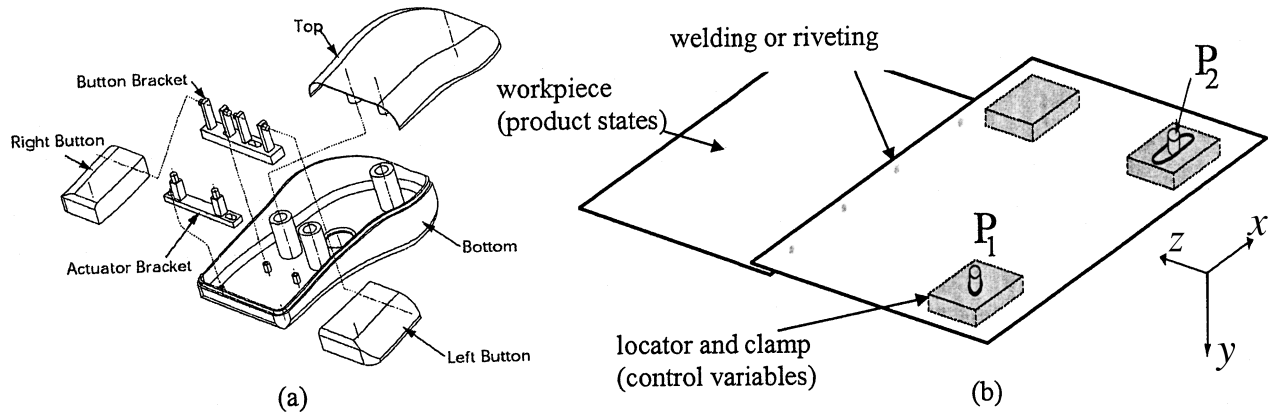


Fig. 4. (a) Type-I assembly; and (b) type-II assembly.

1995; Cunningham *et al.*, 1996). Tolerance analysis and synthesis for tooling elements in type-II assembly processes is another example of process-oriented tolerancing. The reasons that a shift to process-oriented tolerancing from the traditional product-oriented tolerancing is desirable lie in the following aspects:

1. Variations in process variables are root causes of product quality-related problems. Process variables are controllable factors in manufacturing process. Process-oriented tolerancing exerts a direct control on these major variation contributors. Tooling elements that are represented by process variables can potentially be connected with real-time minimum-variance controllers during production to achieve a higher performance.
2. The proposed process-oriented tolerancing is based on a generic mathematical model of variation propagation in multi-station assembly processes. It provides a more systematic and unified approach to different types of manufacturing processes. In contrast, the tolerance charting approach is based on a graphical description and it is mainly developed for stock-removal processes. The model framework of tolerance charting is not generic enough to expand tolerance charting to other part fabrication processes, for instance, progressive (multi-station) sheet metal stamping.
3. The direct inclusion of process variables into tolerance models can potentially lead to the integration of tolerancing with reliability analysis and process maintenance strategies. Process variables provide stochastic information about process dynamics, for example, tooling wear-out. The proposed process-oriented tolerancing can incorporate tooling wear-out variables and lead to a life-cycle tolerance design. Although the effect of tool wear on product quality has been previously studied for machining processes (Quesenberry, 1988; Jensen and Vardeman, 1993; Fraticelli *et al.*, 1999), the discussion in Section 2.4 shows that process-oriented tolerancing requires a different approach.

#### 1.4. Research challenges in process-oriented tolerancing

Process-oriented tolerancing is largely under-investigated even though the variation of process variables has a direct and significant effect on product quality in both part fabrication and assembly processes. The diversity of process variables and the associated complexity is one of the reasons that process variables are seldom explicitly included in tolerancing schemes. Given so many sources of manufacturing process errors, at first it seems infeasible to directly study the tolerances for various process variables. In this paper we utilize a state-of-the-art development in the identification and analysis of variation sources in manufacturing processes, especially in machining and type-II assembly processes (Slocum, 1992; Soons *et al.*, 1992; Cai *et al.*, 1996; Mou, 1997). Those developments are the driving forces behind the proposed process-oriented tolerancing.

Another technical challenge in performing process-oriented tolerancing results from the complex nature of variation propagation in multi-station or multi-stage operations. The variation propagation is conceptually similar to the traditional tolerance stack-up but is generally much more complicated. For instance, if compliant parts are involved in an assembly process, the product variation level could even decrease when the rigidity of a compliant-assembly increases. Generally, we need to identify: (i) the variation transmitted from tooling elements to a product on individual stations; and (ii) the variation induced when the intermediate product is transferred between stations. In order to provide a unified framework that makes the process-oriented tolerancing applicable to different multi-station manufacturing processes, research is required to enable us to systematically model the propagation of variation in a multi-station process. We utilize the matrix perturbation theory previously developed in robotics research (Veitschegger and Wu, 1986; Whitney *et al.*, 1994) to describe complex variation transmissions and adopt a state space representation to recursively represent station-to-station variation propagation. A few multi-station variation propagation models have been developed for rigid-part

assembly processes (Jin and Shi, 1999; Lawless *et al.*, 1999; Mantripragada and Whitney, 1999; Ding *et al.*, 2000), compliant-part assembly process (Camelio *et al.*, 2001), machining processes (Agrawal *et al.*, 1999; Djurdjanovic and Ni, 2001), and stretch forming processes (Suri and Otto, 1999). With these developments, process-oriented tolerancing can be extended to various general multi-station manufacturing processes under a unified framework.

The major contribution of this paper resides in two aspects: we conceptually extend the scope of tolerancing research to explicitly include process variables in tolerancing schemes and develop a tolerance synthesis method for process life-cycle design in a multi-station assembly system with different fixture setups rather than for a single station with one-time fixture set-up. We consider our research effort as one of the initial yet important steps in addressing the general issue of process-oriented tolerancing in multi-station manufacturing processes.

The outline of the paper is as follows. In Section 2, the general framework of process-oriented tolerance synthesis is presented and the detailed models are derived for a multi-station assembly process. Section 3 illustrates the proposed technique using an industrial case study of an automotive body assembly process. Finally, the methodology is summarized in Section 4.

## 2. Process-oriented tolerancing in multi-station assembly

### 2.1. Overview

In the Introduction, we illustrated the difference between process variables and product variables and made the distinction between product-oriented tolerancing and process-oriented tolerancing. Following the manufacturing process flow shown in Fig. 1, process-oriented tolerancing can have three scenarios presented below with a short analysis:

1. *Process-oriented tolerancing of part fabrication processes*: Research work under this scenario has been conducted for machining processes by Rong and Bai (1996) and Choudhuri and DeMeter (1999). However, in their papers, only tolerance analysis (variation simulation) at a *single workstation* with a *one-time tool setup* is discussed. The proposed process-oriented tolerancing model includes not only tolerance analysis but also tolerance synthesis in multi-station processes with multiple tool set-ups.
2. *Process-oriented tolerancing of assembly processes*: Existing papers on tolerancing for type-II assembly (Liu *et al.*, 1996; Ceglarek and Shi, 1997) have focused on the effect of flexibility in compliant-parts on tolerance analysis. Effects from tooling elements were not included in their study. Commercial software packages such as 3-DCS (Anon, 2000) and VSA (Anon, 1998) can perform variation simulation in the forward direction. The tolerance synthesis as the inverse problem is difficult for

a simulation software to solve. The tolerance synthesis needs an analytical model to describe the propagation of variation in a multi-station process.

3. *Process-oriented tolerancing for an integrated fabrication and assembly process*: The third scenario is an integration of the above two scenarios, i.e., simultaneously allocate tolerances to tooling elements in the assembly process and process variables in the part fabrication process. There have been efforts to integrate the allocation of design tolerances and manufacturing tolerances (both for the product variables) in stages 1 and 2 (Zhang, 1996). Our proposed method possesses a similar philosophy but extends the scope of tolerancing to process variables.

The detailed development presented in this paper is focused on the second scenario, i.e., how to optimally allocate tolerances to fixture elements in a multi-station assembly process. The choice is made based on the understanding that little research has been reported in this sub-area of process-oriented tolerancing. However, since we employ a general state space approach in modeling variation propagation of multi-station processes, the proposed approach can be readily extended to many different processes such as machining or stamping processes.

A schematic diagram is shown in Fig. 5 to demonstrate the interrelations between tolerances, quality, and cost in a multi-station manufacturing process. A cost is associated with the tolerances assigned to both the *process* and *product variables*. Variations in these variables, determined by their tolerances, will affect the quality of the final product. The variation in process variables caused by different manufacturing stations is the focus of this paper. The variation of product variables, i.e., the variation of each component/part resulted from precedent processes, is treated as an initial variation condition for the current assembly process.

As we pointed out in the Introduction, process variables provide dynamic process information (say, information related to a tool wear process). Thus, process variables are strongly related to process reliability and the corresponding maintenance policies. If tolerances are allocated without considering tooling degradation, product quality can only be guaranteed at the very initial stage of a production. However, quality criteria should be satisfied not only during the initial stage of a production but also during the whole life-cycle of a production system. Currently, in numerous real production systems, maintenance is conducted based on a fixed time schedule. For example, all locating pins at assembly stations are replaced every half year. In this case, the initial tolerances need to be tighter to accommodate tooling degradation between scheduled maintenance actions so as to avoid out-of-specification products. Mathematically, the optimal tolerance  $\mathbf{T}^*$  can be formulated as the following constrained optimization:

$$\mathbf{T}^* = \arg \min_{\mathbf{T}} C_{\mathbf{T}}(\mathbf{T}) \quad (1)$$

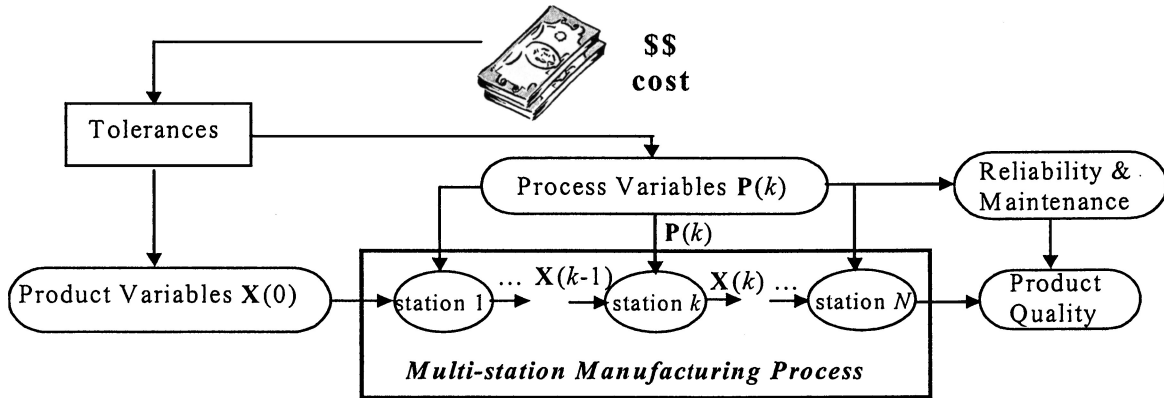


Fig. 5. An overview of the process-oriented tolerance synthesis.

subject to

$$g(\mathfrak{S}_Q(\mathbf{T}, t), C) \geq 0 \quad \text{for } \{t: 0 < t < t_m\},$$

where  $C_T$  represents the cost function,  $\mathbf{T}$  is the tolerance vector of selected key process variables,  $g(\cdot, \cdot)$  is a constraint function to be determined from the nature of a quality measure,  $\mathfrak{S}_Q(\cdot, \cdot)$  is a given measure or index of product quality,  $C$  is the threshold of specified product quality,  $t$  is the time, and  $t_m$  represents the maintenance time period.

The cost function ( $C_T$ ) is determined by tolerances assigned to the process variables. Generally, the tighter the tolerance, the higher the cost of satisfying it. The reciprocal function and negative exponential function are widely used as cost functions (Wu *et al.*, 1988). However, process variables are very diversified and are not limited to describing geometrical relations, which may create difficulty in selecting the most appropriate cost function for the tolerance. The choice of the cost function for the process variable strongly depends on the physics of that variable. The selection of a cost function for a non-geometrical variable is not discussed in this paper.

The second question is how to relate the tolerances ( $\mathbf{T}$ ) to the product quality index ( $\mathfrak{S}_Q(\cdot, \cdot)$ ), which is part of the constraint function ( $g(\cdot, \cdot)$ ). The development of such a constraint function needs several essential models as is shown in Fig. 6. Tolerances are first related to the variations in process variables. A product variation-stream propagates along a production line with variable contributions that accumulate at each station. Eventually, a proper measure is exerted to compare product variation with a specified product quality index. Overall, there are four key elements to realize the above optimization formulation: (i) a variation propagation model; (ii) a tolerance-variation relation; (iii) a process degradation model; and (iv) a cost function.

2.2. State space model of variation propagation

Multi-station assembly processes such as an automotive body assembly are described in detail in Ceglarek *et al.* (1994). The modeling of fixture-related variation propagation in such an assembly process has been studied by Shiu *et al.* (1996), Jin and Shi (1999), and Ding *et al.* (2000). Two major variation contributors have been identified: (i) fixture-induced variation at each single station caused by

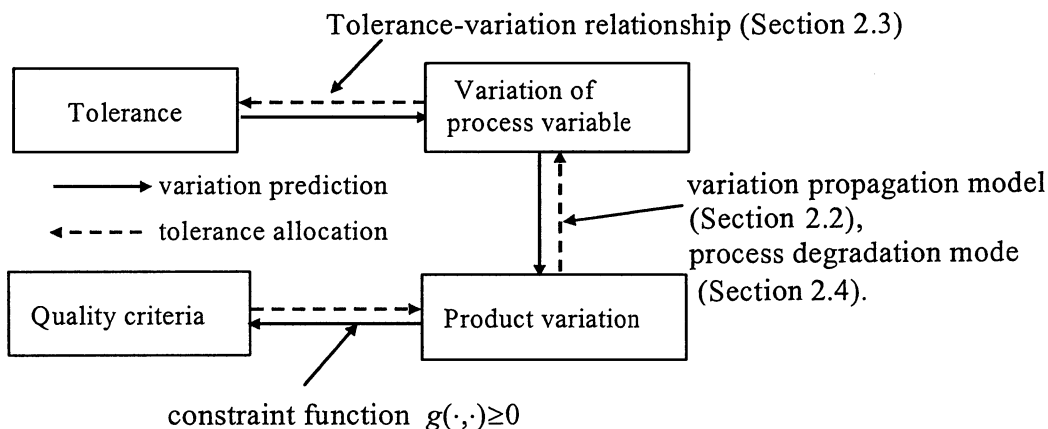


Fig. 6. The relationship between tolerance and quality.

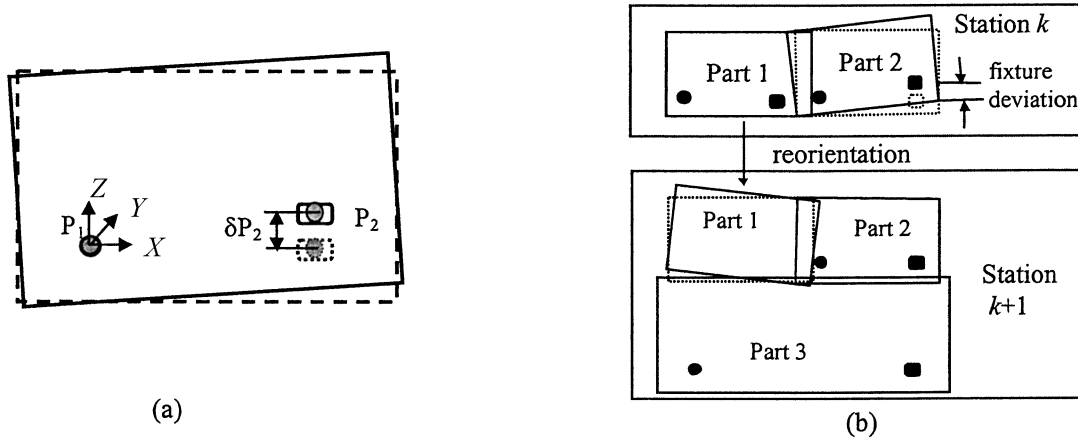


Fig. 7. Variation induced at: (a) single station; and (b) across stations.

fixture locators failures (Fig. 7(a)); for example for a 3-2-1 fixture layout with two locators  $P_1$  and  $P_2$ , the failure of locator  $P_2$  represented as  $\delta P_2(z)$  is the part deviation in Z-direction at locator  $P_2$ ; and (ii) the reorientation-induced variation caused by possible locating layout change between stations (Fig. 7(b)). The first factor, the fixture-induced variation at each station, is affected by the geometry of the fixture locating layout, i.e., the coordinates of the fixture locators. The second factor, the reorientation-induced variation, is affected by the magnitude of the fixture locating layout changes between stations.

These two variation contributions and their propagation can be modeled in a  $N$ -station assembly process as is shown in Fig. 8 by using a state space representation (Jin and Shi, 1999; Ding *et al.*, 2000). The basic idea in developing the state space variation model is to consider the multi-station process as a sequential dynamic system but to replace the time index in the traditional state space model with a station index. The state space model consists in two equations:

$$\mathbf{X}(k) = \mathbf{A}(k-1)\mathbf{X}(k-1) + \mathbf{B}(k)\mathbf{P}(k) + \mathbf{W}(k), \quad (2)$$

$$\mathbf{Y}(k) = \mathbf{C}(k)\mathbf{X}(k) + \mathbf{V}(k), \quad (3)$$

where the first equation, known as the state equation, suggests that the part deviation at station  $k$  is influenced by the

accumulated deviation up to station  $k-1$  and the deviation contribution at station  $k$ ; the second equation is the observation equation.

In the above equations,  $\mathbf{X}(k)$  is the product quality information (e.g., part dimensional deviations) after operations at station  $k$ ,  $\mathbf{P}(k)$  is the process variation contributed at station  $k$ , product measurements at KPCs at station  $k$  are included in  $\mathbf{Y}(k)$ , and  $\mathbf{W}(k)$  and  $\mathbf{V}(k)$  are unmodeled errors and sensor noises, respectively. Matrices  $\mathbf{A}(k)$  and  $\mathbf{B}(k)$  include information regarding process design such as fixture layout on individual stations and the change of fixture layouts across stations, and  $\mathbf{C}(k)$  includes sensor deployment information (the number and location of sensors on station  $k$ ). The corresponding physical interpretation of  $\mathbf{A}$ ,  $\mathbf{B}$ , and  $\mathbf{C}$  is presented in Table 1, where  $\Phi(k, j) \equiv \mathbf{A}(k-1) \cdots \mathbf{A}(j)$  and  $\Phi(j, j) \equiv \mathbf{I}$  ( $\mathbf{I}$  is an identity matrix with appropriate dimensions) and the detailed expression can be found in Jin and Shi (1999) and Ding *et al.* (2000).

Suppose that there is only an end-of-line observation, that is,  $k = N$  in the observation equation of Equation (3). Then, we have:

$$\mathbf{Y}(N) = \sum_{k=1}^N \mathbf{C}(N)\Phi(N, k)\mathbf{B}(k)\mathbf{P}(k) + \mathbf{C}(N)\Phi(N, 0)\mathbf{X}(0) + \boldsymbol{\varepsilon}. \quad (4)$$

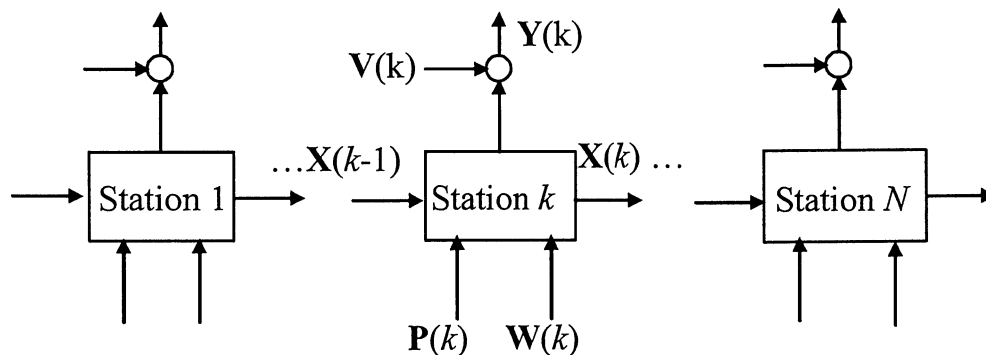


Fig. 8. Diagram of an assembly process with  $N$  stations.

**Table 1.** Interpretation of system matrices

Symbol	Name	Relationship	Interpretation	Assembly task
A	Dynamic matrix	$\mathbf{X}(k-1) \xrightarrow{\mathbf{A}(k-1)} \mathbf{X}(k)$	Change of fixture layout between two adjacent stations	Assembly transfer
$\Phi(k, i)$	State transition matrix	$\mathbf{X}(i) \xrightarrow{\Phi(k, i)} \mathbf{X}(k)$	Change of fixture layout among multiple stations	Assembly transfer
B	Input matrix	$\mathbf{P}(k) \xrightarrow{\mathbf{B}(k)} \mathbf{X}(k)$	Fixture layout at station $k$	Part positioning
C	Observation matrix	$\mathbf{X}(k) \xrightarrow{\mathbf{C}(k)} \mathbf{Y}(k)$	Sensor layout at station $k$	Inspection

Here,  $\mathbf{X}(0)$  corresponds to the initial condition, that is a result of imperfect manufacturing of stamped parts, and  $\boldsymbol{\varepsilon}$  is the summation of all modeling uncertainty and sensor noise terms (modeled in terms of  $\mathbf{W}$  and  $\mathbf{V}$ ). Moreover, it was assumed that this process involves sheet metal assembly with only lap-lap joints and thus, the stamping imperfection of part dimensions will not affect the propagation of variations. Then, we can set the initial conditions to zero. The uncertainty term  $\boldsymbol{\varepsilon}$  can be neglected in the design stage given the fact that a simulation study presented in Ding *et al.* (2000) showed that the uncertainty term  $\boldsymbol{\varepsilon}$  accounts for a 0.02% extra variation in a standard three-station automotive body assembly process with 3-2-1 fixtures. The variation propagation can then be approximated as:

$$\mathbf{K}_Y = \sum_{k=1}^N \gamma(k) \mathbf{K}_P(k) \gamma^T(k), \quad (5)$$

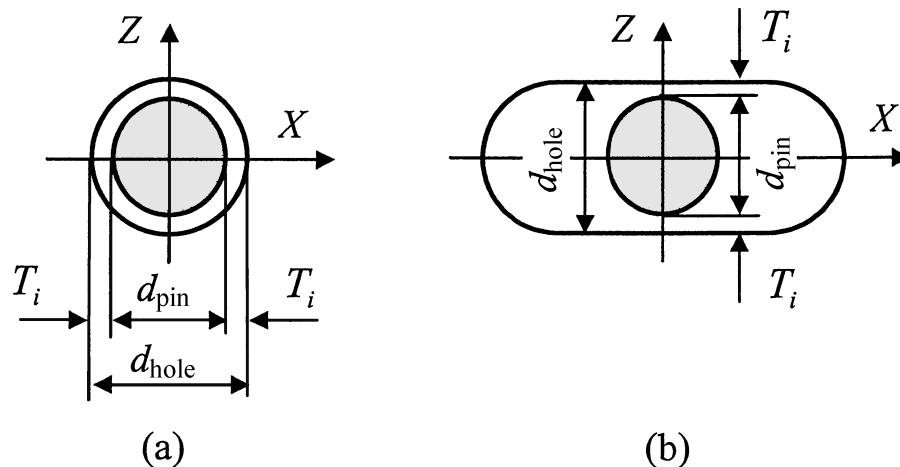
where  $\mathbf{K}_Y$  and  $\mathbf{K}_P(k)$  represent the covariance matrices of  $\mathbf{Y}(N)$  and  $\mathbf{P}(k)$ , respectively, and  $\gamma(k) \equiv \mathbf{C}(N) \Phi(N, k) \mathbf{B}(k)$ . Based on engineering knowledge, it is known that the process variable in this problem is the fixturing error at every assembly station, which is often caused by the clearance of locating pin-hole pairs.

### 2.3. Relationship between the tolerance and variation

The presented analysis is conducted for part fixturing based on a pin-hole type of locator, which is commonly used in automotive assembly processes. However, a similar analysis can be conducted for other part fixturing locating elements used in different processes. There are two major types of pin-hole locating pair: (1) a four-way pin-hole locating pair; and (2) a two-way pin-hole locating pair, shown in Fig. 9(a and b), where  $d_{\text{pin}}$  or  $d_{\text{hole}}$  is the diameter of a pin or a hole and  $T_i$  is the specified tolerance of  $i$ th clearance, that is, the upper limit of the  $i$ th clearance.

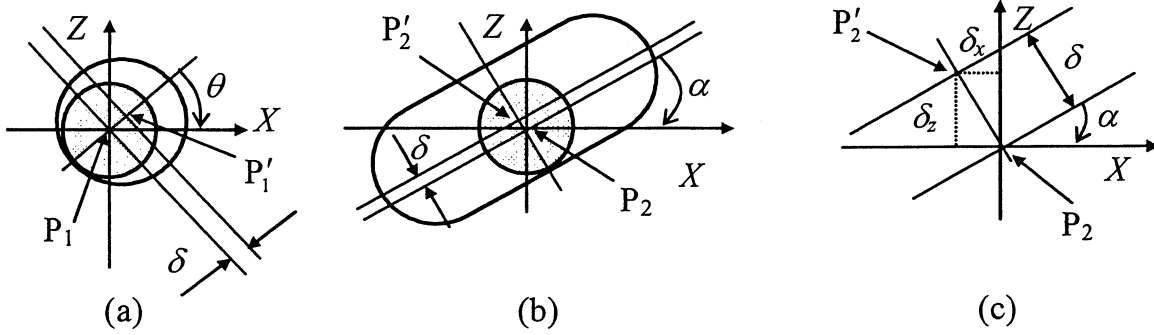
A four-way pin-hole locating pair includes a homogeneous circular hole and controls the motion in both the  $X$  and  $Z$  directions (Fig. 9(a)). A two-way pin-hole locating pair consists in a slot and a circular pin and thus only controls the motion perpendicular to the long axis of the slot, i.e., the  $Z$  direction in Fig. 9(b). These two types of locating pairs are used together to position a part during assembly. Due to the free motion along the  $X$  axis of a two-way pin-hole locating pair, the part is not over-constrained in the fixture.

Our primary interest is to study the variations associated with a pin-hole locating pair caused by its clearance. The clearance-induced deviation is shown in Fig. 10(a–c). Its geometrical relationship is obtained by Jin and Chen (2001). The deviation of a four-way locating pair is exemplified in Fig. 10(a), in which the deviation of  $P'_1$  (the center of the



**Fig. 9.** Diagram of the pin-hole locating pairs: (a) four-way pin-hole pair; and (b) a two-way pin-hole pair.





**Fig. 10.** Clearance-induced deviation: (a) for a four-way locating pair; and (b) and (c) for a two-way locating pair with an orientation angle of  $\alpha$ .

pin-hole pair) from  $P_1$  (the center of the pin) in both the  $X$  and the  $Z$  directions are:

$$\Delta X = \delta \cos \theta, \quad (6)$$

$$\Delta Z = \delta \sin \theta, \quad (7)$$

where  $\delta$  is the distance between  $P'_1$  and  $P_1$  and  $\theta$  is the contact orientation.

Denote by  $\delta$  the random variable representing the actual clearance in one setup. Then  $\delta$  is bounded by  $[0, T_i]$  since  $T_i$  is the clearance tolerance. We approximate  $\delta$  by a normal distribution:

$$N\left(\frac{T_i}{2}, \left(\frac{T_i}{6}\right)^2\right).$$

The clearance of a four-way locating pair is considered to be homogenous in all directions and thus the orientation angle  $\theta$  is of a uniform distribution between 0 and  $2\pi$ , i.e.,  $\theta \sim U(0, 2\pi)$ . Given that  $\delta$  and  $\theta$  are independent of one another, the statistics regarding  $\Delta X$  and  $\Delta Z$  are given as:

$$E[\Delta X] = E[\delta \cos \theta] = E[\delta] \times E[\cos \theta] = 0, \quad (8)$$

$$E[\Delta Z] = E[\delta \sin \theta] = E[\delta] \times E[\sin \theta] = 0, \quad (9)$$

$$\sigma_{X,4\text{-way}}^2 = E[\Delta X^2] = E[\delta^2] \times E[\cos^2 \theta] = \frac{5T_i^2}{36}, \quad (10)$$

$$\sigma_{Z,4\text{-way}}^2 = E[\Delta Z^2] = E[\delta^2] \times E[\sin^2 \theta] = \frac{5T_i^2}{36}, \quad (11)$$

$$\begin{aligned} \text{Cov}(\Delta X, \Delta Z) &= E[\Delta X \Delta Z] = E[\delta^2 \sin \theta \cos \theta] \\ &= E[\delta^2] \times E[\sin \theta \cos \theta] = 0, \end{aligned} \quad (12)$$

where  $E[\cdot]$  is the expectation operator and  $\text{Cov}(\cdot, \cdot)$  represents the covariance of two random variables. These equations suggest that the deviations of  $P'_1$  in both directions have a zero mean and the same variances. They are uncorrelated according to Equation (12).

The geometrical relationship of a two-way locating pair with an orientation angle of  $\alpha$  shown in Fig. 10 (b and c) is:

$$\delta_X = \delta \sin \alpha \times \kappa \quad \text{and} \quad \delta_Z = -\delta \cos \alpha \times \kappa, \quad (13)$$

where  $\delta$  is defined in the same way as before and  $\kappa$  is a binary random variable with its value being either 1 or  $-1$ . We postulate that if the pin touches the top (or left if  $\alpha$  approaches  $90^\circ$ ) edge of a pin-hole, then  $\kappa$  is one; if the pin touches the bottom (or right if  $\alpha$  approaches  $90^\circ$ ) edge of a pin-hole, then  $\kappa$  is  $-1$ . Also,  $\kappa$  is independent of  $\delta$ . Hence, the variation associated with a two-way locating pair can be expressed as:

$$E[\delta_X] = E[\delta_Z] = 0, \quad (14)$$

$$\sigma_{X,2\text{-way}}^2 = E[\delta^2 \sin^2 \alpha \times \kappa^2] = \frac{5T_i^2}{18} \sin^2 \alpha, \quad (15)$$

$$\sigma_{Z,2\text{-way}}^2 = E[\delta^2 \cos^2 \alpha \times \kappa^2] = \frac{5T_i^2}{18} \cos^2 \alpha, \quad (16)$$

$$\text{Cov}(\delta_X, \delta_Z) = E[\delta^2 \cos \alpha \sin \alpha \times \kappa^2] = \frac{5T_i^2}{18} \cos \alpha \sin \alpha. \quad (17)$$

Equation (17) implies that the deviations of a two-way locating pair at an arbitrary orientation angle  $\alpha$  are correlated. Equations (10), (11), (15), and (16) will be iteratively applied to every pin-hole locating pair on each station in a multi-station assembly process so that  $\mathbf{K}_P(k)$  can be expressed in terms of corresponding fixture tolerances.

*Remark 1.* The model of pin-hole contact discussed here can be considered as a special case of the chain-link models presented in BJORKE (1989) where the two-way pin-hole contact is the lumped-magnitude/lumped-direction case and the four-way pin-hole is the lumped-magnitude/distributed-direction case. For some other manufacturing processes other models may be required (for example, distributed-magnitude/distributed-direction as mentioned in BJORKE (1989)); which need to be developed separately from the analysis presented in this paper.

*Remark 2.* The pin-hole contact in a fixture locating scheme may resemble the geometrical relationship of a shaft-hole contact between parts in an assembly-product. However, the difference is that the clearance in a pin-hole contact is not a product variable but a process variable because the locating pin is not a part of the product.

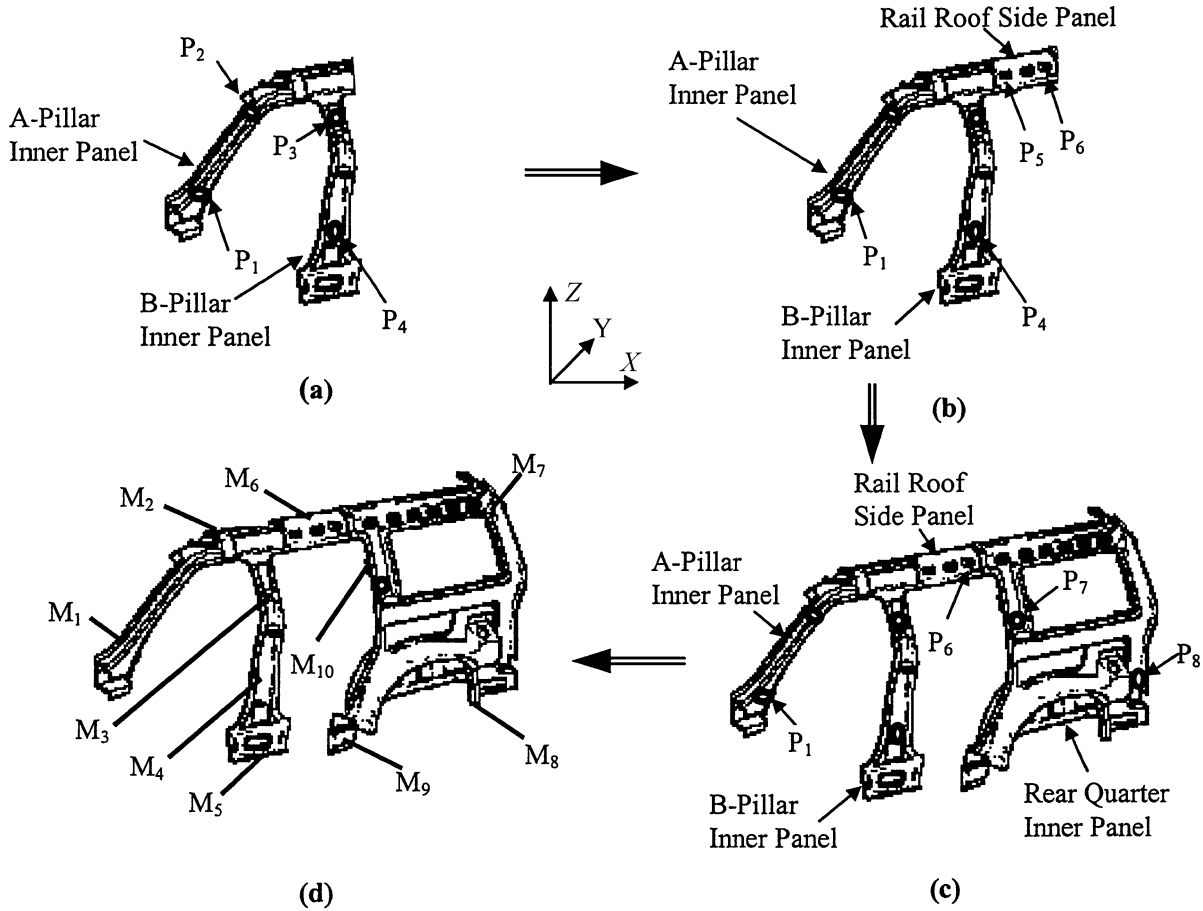


Fig. 11. Side aperture inner panel assembly: (a) station I; (b) station II; (c) station III; and (d) measurement: KPC points.

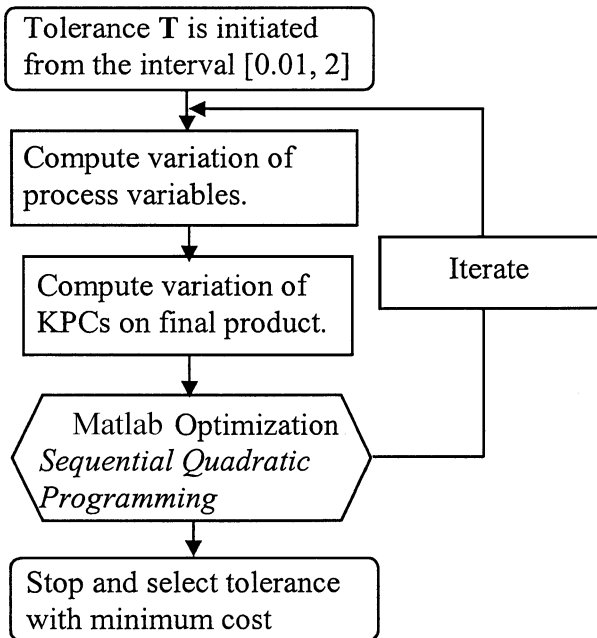


Fig. 12. Tolerance allocation without a degradation model.

2.4. Process degradation model

The process degradation considered in this assembly process is caused by a locator wear process. The effects of tool wear have been considered in the literature. Fainguelernt *et al.* (1986) treated tool wear as a static equivalent error and allocated a tolerance to satisfy a worst-case requirement. Quesenberry (1988) and Jensen and Vardeman (1993) did not address the tolerance issue but instead considered how to compensate the tool wear effect by utilizing in-line observations. Fraticelli *et al.* (1999) used Sequential Tolerancing Control (STC) to compensate for the random error that results from tool wear. However, given the fact that the in-line observations were obtained after the tool wear actually occurred, what was considered in STC is a realization of the stochastic tool wear process rather than the true stochastic process itself.

If the tool wear is severe, as in the case of machining processes, a frequent compensation or machine tool recalibration is necessary. In such a situation, a process control strategy using the above adjustment mechanism or STC is recommended. In assembly processes, tool wear is a relatively slow process and its effect on product quality will only be manifested after a substantial accumulation time. In

**Table 2.** Coordinates of fixture locators from Fig. 11(a–d) (unit: mm)

	$P_1$	$P_2$	$P_3$	$P_4$	$P_5$	$P_6$	$P_7$	$P_8$
(X, Z)	(367.8, 906.05)	(667.47, 1295.35)	(1301, 1368.89)	(1272.73, 537.37)	(1470.71, 1640.40)	(1770.50, 1702.62)	(2941.42, 1691.31)	(2120.32, 1402.83)

assembly processes, we can use the initial tolerance range to accommodate the randomness generated from a tool wear process. The inclusion of stochastic tool wear phenomena in the tolerancing scheme is one of the features of process-oriented tolerancing.

The sliding wear model which serves as the governing physical mechanism of tool wear processes was first studied by Archard (1953), where the incremental wear was characterized by:

$$\Delta_r = k_w \frac{Lx}{A}, \quad (18)$$

where  $L$  is the loading,  $x$  is the slide distance,  $A$  is the size of contact area, and  $k_w$  is a random coefficient. Wallbridge and Dowson (1987) and Tang *et al.* (1988) concluded that the coefficient  $k_w$  has a log-normal distribution that is determined by material properties and sliding conditions. Tang *et al.* (1988) gave the mean value of  $k_w$  for alloy-steel materials in a moderate sliding wear condition as  $5 \times 10^{-5} \text{ mm}^3 \text{ N}^{-1} \text{ m}^{-1}$ . The other parameters in Equation (18) are determined by engineering measurements and estimations of actual sliding pairs. Using these basic models, Jin and Chen (2001) established a stochastic degradation model for the tool wear process. The tool wear aggregates when the number of operations increases. The aggregated wear  $\Delta_d(t)$  at operation  $t$  is expressed as:

$$\Delta_d(t) = \Delta_d(t - 1) + \Delta_r(t), \quad (19)$$

where  $\Delta_r(t)$  is the incremental wear due to operation  $t$ . Since  $k_w$  has a log-normal distribution,  $\Delta_r(t)$  also has a log-normal distribution, i.e.,  $\Delta_r(t) \sim \text{Lognorm}(\mu_\Delta(t), \sigma_\Delta^2(t))$ . The mean wear-out rate  $\mu_\Delta$  consists of two components, a constant wear-out rate plus a higher initial wear-out rate that decreases exponentially. The mean wear-out rate for operation  $t$  is assumed to be:

$$\mu_\Delta(t) = \mu_0 + \mu_1 e^{-\beta t}, \quad (20)$$

where  $\mu_0 + \mu_1$  is the initial wear-out rate,  $\mu_0$  is the constant rate, and  $\beta$  determines how fast the wear-out will reach its steady-state value. The clearance change of a pin-hole locating pair can be computed by:

$$d(t) = \delta + \Delta_d(t), \quad (21)$$

where  $d(t)$  is the clearance after operation  $t$  and  $\delta$  is the initial clearance which is the same as that in Equations (6) and (7). This implies that the clearance increases after a pin wears out and that the locating variation also increases. We should substitute the enlarged clearance at time  $t_m$  into Equations (6), (7), and (13) and recalculate the locating variation. In the next derivations, we make the following assumptions: (i) the initial clearance  $\delta$ , the orientation variables  $\theta$  and  $\kappa$ , and the aggregated wear  $\Delta_d(t)$  are assumed to be independent of one another; (ii) the variance of the wear-out rate  $\sigma_\Delta^2$  is assumed to be the same for all operations; and (iii) according to the central limit theorem, the aggregated wear  $\Delta_d(t)$  can be approximated by a normal distribution after a large enough number of operations. Based on these properties and assumptions, the following relationships can be obtained by substituting Equation (21) into Equations (11), (12), (16) and (17), respectively:

$$\begin{aligned} \sigma_{X,4\text{-way}}^2(t_m) &= E[(\delta + \Delta_d(t_m))^2 \times \cos^2 \theta] \\ &= \frac{1}{2} E[(\delta + \Delta_d(t_m))^2], \end{aligned} \quad (22)$$

$$\begin{aligned} \sigma_{Z,4\text{-way}}^2(t_m) &= E[(\delta + \Delta_d(t_m))^2 \times \sin^2 \theta] \\ &= \frac{1}{2} E[(\delta + \Delta_d(t_m))^2], \end{aligned} \quad (23)$$

$$\begin{aligned} \sigma_{X,2\text{-way}}^2(t_m) &= E[(\delta + \Delta_d(t_m))^2 \sin^2 \alpha \times \kappa^2] \\ &= \sin^2 \alpha \times E[(\delta + \Delta_d(t_m))^2], \end{aligned} \quad (24)$$

$$\begin{aligned} \sigma_{Z,2\text{-way}}^2(t_m) &= E[(\delta + \Delta_d(t_m))^2 \cos^2 \alpha \times \kappa^2] \\ &= \cos^2 \alpha \times E[(\delta + \Delta_d(t_m))^2], \end{aligned} \quad (25)$$

where

$$\begin{aligned} E[(\delta + \Delta_d(t_m))^2] &= E[\delta^2 + 2\delta\Delta_d(t_m) + \Delta_d^2(t_m)], \\ &= E[\delta^2] + 2E[\delta]\bar{d}(t_m) + \text{Var}(\Delta_d(t_m)) \\ &\quad + \bar{d}(t_m)^2, \\ &= \frac{5T_i^2}{18} + T_i\bar{d}(t_m) + t_m\sigma_\Delta^2 + \bar{d}(t_m)^2, \\ &= \frac{5}{18} \left( T_i + \frac{9}{5}\bar{d}(t_m) \right)^2 + t_m\sigma_\Delta^2 \\ &\quad + \frac{1}{10}\bar{d}(t_m)^2, \end{aligned} \quad (26)$$

and  $\bar{d}(t_m) = E[\Delta_d(t_m)]$  is the average aggregated wear.

**Table 3.** Coordinates of KPCs from Fig. 11(d) (unit: mm)

KPC	$M_1$	$M_2$	$M_3$	$M_4$	$M_5$	$M_6$	$M_7$	$M_8$	$M_9$	$M_{10}$
(X, Z)	(271.50, 905)	(565.7, 1634.7)	(1289.7, 1227.5)	(1306.5, 633.5)	(1244.5, 85)	(1604.5, 1781.8)	(2884.8, 1951.5)	(2743.5, 475.2)	(1838.4, 226.3)	(1979.8, 1459.4)

**Table 4.** Tolerances without tooling degradation (unit: mm)

$T_1$	$T_2$	$T_3$	$T_4$	$T_5$	$T_6$	$T_7$	$T_8$	$T_9$	$T_{10}$	$T_{11}$	$T_{12}$
0.21	0.36	0.19	0.31	0.30	0.42	0.63	0.36	0.34	0.34	0.34	0.32

### 2.5. Cost function

Various cost functions for the tolerance have been proposed for different tolerance synthesis schemes by Wu *et al.* (1988) who proposed five types of cost-tolerance functions. It was found that the Michael-Siddal function was the best fit to the actual data and the reciprocal squared function the worst fit. However, the Michael-Siddal function is a complex function with too many parameters to be determined. The exponential function and the reciprocal function are good alternatives with a decent data fit and simple function structure. In this paper, we select the reciprocal function as the cost-tolerance function due to its simplicity. That is:

$$C_T = \sum_{i=1, \dots, n_T} \frac{w_i}{T_i}, \quad (27)$$

where  $T_i$  is the  $i$ th tolerance,  $i = 1, 2, \dots, n_T$  and  $w_i$  is the weight coefficient associated with  $T_i$ . However, the exponential function can also be used and the general conclusions regarding the optimal procedure and optimality will not change. In the above equation,  $w_i$  determines the relative importance or the relative manufacturing cost associated with each tolerance to be allocated. The relative importance or cost will be determined by engineering design knowledge or engineering accounting practice.

### 2.6. Optimization formulation and optimality

Many optimization schemes including linear optimization, nonlinear optimization, integer optimization, and genetic algorithms have been studied in traditional tolerancing techniques (Ashiagbor *et al.* 1998; Lee and Woo, 1989, 1990).

In our problem, once the essential process models studied in Sections 2.2–2.5 are available, a constrained nonlinear optimization problem is formulated for the multi-station assembly process as:

$$\mathbf{T}^* = \min_{\mathbf{T}} \text{arg}\{C_T(\mathbf{T})\} \quad (28)$$

subject to

$$g(\mathfrak{S}_Q, \sigma_s^2) = \sigma_s^2 - \|\text{diag}(\mathbf{K}_Y)\|_{\infty} \geq 0 \quad \text{for all } 0 < t < t_m \\ \text{and } T_i > 0 \quad \forall i$$

where the quality measure is  $\mathfrak{S}_Q(\mathbf{T}, t) = \|\text{diag}(\mathbf{K}_Y)\|_{\infty}$  that extracts the diagonal elements of  $\mathbf{K}_Y$ , i.e.,  $\text{diag}(\mathbf{K}_Y)$  includes the variances of the KPC points on the final product. The current choice of constraint function requires that the variations of all KPCs on the final product must be less than a given upper variation limit (i.e.,  $\sigma_s^2$  in this formulation). This constraint function is only one of many possible choices,

corresponding to a criterion currently used in industrial practice. Other valid measures such as 1-norm and 2-norm may also be used in the constraint function. The use of a  $\infty$ -norm is consistent with the Pareto principle in quality engineering, i.e., the quality requirement is imposed on the KPCs with relatively large variation values. Our industrial experience indicates that the use of a  $\infty$ -norm is more easily accepted by industrial practitioners.

It can be also concluded (based on theorems in Zangwill (1967)) that Equation (28) achieves a global optimality because the cost-function is convex and the constraint function is a concave quadratic function. Any available nonlinear programming software package can be used to solve this optimization problem.

### 3. Example

The automotive assembly process of a side aperture inner panel is used to illustrate the tolerancing procedure of a multi-station process. This assembly process, shown in Fig. 11(a–d) is conducted at three assembly stations (stations I, II and III) and the product is inspected at the measurement station. The final subassembly *inner-panel-complete* (Fig. 11(c)) consists of four components: (i) A-pillar inner panel; (ii) B-pillar inner panel; (iii) rail roof side panel; and (iv) rear quarter inner panel. At station I (Fig. 11(a)), the A-pillar inner panel and the B-pillar inner panel are joined together. The subassembly “A-pillar + B-pillar” is welded with the rail roof side panel at station II (Fig. 11(b)). The subassembly of the first three panels is then assembled with the rear quarter inner panel at station III (Fig. 11(c)). Finally, measurements are taken at KPC points (marked in Fig. 11(d) as  $M_1 - M_{10}$ ) by using either off-line or in-line measurement systems such as CMM or OCMM. The nominal design positions of the fixture locators and KPC points in 2-D ( $X$ – $Z$  coordinates) are given in Table 2 and Table 3, respectively.

Before conducting process-oriented tolerancing, we need to establish a state space variation model for this particular panel assembly process. This process has  $N = 4$ . Since the fixture used on the inspection station is considered to

**Table 5.** Parameters in the degradation model

$\mu_0$ (mm)	$\mu_1$ (mm)	$\beta$	$\sigma_{\Delta}$ (mm)	$t_m$	Operations/day
$5 \times 10^{-7}$	$1 \times 10^{-6}$	$1 \times 10^{-3}$	$5 \times 10^{-5}$	6	500
					months

**Table 6.** Tolerances with tooling degradation (unit: mm)

$T_1$	$T_2$	$T_3$	$T_4$	$T_5$	$T_6$	$T_7$	$T_8$	$T_9$	$T_{10}$	$T_{11}$	$T_{12}$
0.16	0.31	0.14	0.25	0.23	0.34	0.58	0.26	0.30	0.27	0.28	0.26

be well maintained and calibrated with a much higher repeatability than those on a regular assembly station, the input variation of the fixture locators on the measurement station is neglected. The deviation inputs from fixtures on three assembly stations,  $\mathbf{P}(1)$ ,  $\mathbf{P}(2)$ , and  $\mathbf{P}(3)$ , are included. The state space variation model is:

$$\begin{cases} \mathbf{X}(1) = \mathbf{B}(1)\mathbf{P}(1) + \mathbf{W}(1), \\ \mathbf{X}(k) = \mathbf{A}(k-1)\mathbf{X}(k-1) + \mathbf{B}(k)\mathbf{P}(k) + \mathbf{W}(k), \quad k = 2, 3, \\ \mathbf{X}(4) = \mathbf{A}(3)\mathbf{X}(3) + \mathbf{W}(4), \\ \mathbf{Y} = \mathbf{C}\mathbf{X}(4) + \mathbf{V}, \end{cases} \quad (29)$$

where the  $\mathbf{A}$ 's,  $\mathbf{B}$ 's, and  $\mathbf{C}$  can be obtained following the procedure outlined in Ding *et al.* (2000).

**3.1. Tolerance allocation when tooling degradation is not considered**

There are 12 tolerance variables of clearance  $T_1 - T_{12}$  to be allocated in this three-station process (each station has four pin-hole locating pairs). It is assumed that all process variables are subject to the same manufacturing cost, that is,  $w_i = 1$  for  $i = 1, 2, \dots, 12$  in Equation (27). The designer requires that the final product (the inner-panel-complete) must have a six-sigma value less than 1.5 mm at all KPCs, namely  $\sigma_s^2 = (1.5/6)^2$  in Equation (28). From industrial practice, it is known that the tolerance of a clear-

ance is usually larger than 0.01 mm. Thus, the initial tolerance is then picked up from the interval [0.01, 2] mm. The procedure for tolerance allocation is shown in the following flow chart (Fig. 12).

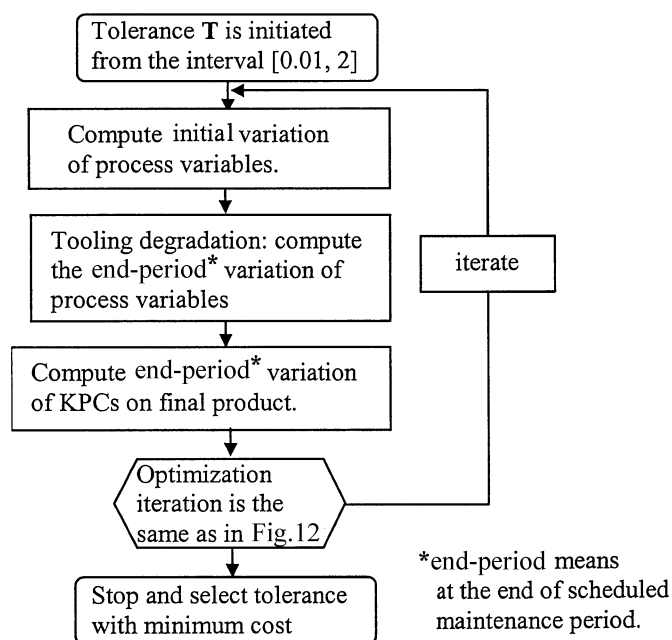
The optimization problem is solved using the Matlab function *fmincon* which uses a Sequential Quadratic Programming (SQP) method (Anon, 1999). The SQP algorithm operates by solving a sequence of quadratic subproblems. Each quadratic subproblem represents solving an approximation to the Langrangian function. The SQP is chosen because it is a very efficient nonlinear programming algorithm and is commercially available. The algorithm can converge to the global optimum due to the quadratic nature of the optimization problem in Equation (28). Due to the availability of analytical models developed in Section 2, a time-consuming Monte Carlo simulation can be avoided when the variation of process variables needs to be obtained. The program converges in minutes and yields the optimal tolerance after 290 iterations. The optimally allocated tolerances for these process variables are listed in Table 4.

Compared with the current industry practice, where the tolerance for a locating clearance is allocated uniformly for all locating pairs, the proposed approach no longer allocates tolerances uniformly. This nonuniformity is consistent with process sensitivity, that is, the more variation a process variable contributes to the final product, the tighter should be the corresponding tolerance. It is difficult for an empirical approach to determine which tolerance should be tight. As a result, either the cost is higher or the variation of the final product is above the threshold using empirical approaches; or in other words, optimality is difficult to achieve.

**3.2. Tolerance allocation with consideration of tooling degradation**

Under this circumstance, tolerances are allocated at the beginning of production and the quality criteria are checked for all products produced by the degraded process. The procedure for tolerance allocation with consideration of tooling degradation is shown in Fig. 13.

The optimization is still solved using the Matlab function *fmincon* but with the tooling degradation model implemented. Based on industry experience, the parameters needed in the degradation model such as operation rate,



**Fig. 13.** Tolerance allocation with a degradation model.

**Table 7.** Maximum  $6\sigma$  of KPCs for a 0.25 mm tolerance

Beginning	Half-year	Specified
$6\sigma = 1.44$ mm	$6\sigma = 1.77$ mm	$6\sigma = 1.50$ mm

**Table 8.** Comparison of manufacturing costs for different scenarios

Conditions	Without degradation	With degradation	Uniform 0.25 mm
Cost	38.1	47.9	48

maintenance period, and pin wear-out rate are listed in Table 5. The program converges and yields the optimal tolerance after almost the same number of iterations as in Section 3.1. The new tolerances become tighter and are listed in Table 6.

### 3.3. Comparison and discussion

In today's automotive industry, tolerances are uniformly set to be 0.25 mm for all clearances. Substituting these tolerances into the system model described in Section 2, the maximum six-sigma values for the KPCs both at the beginning of production and after a half-year of production are listed in Table 7. Although the assigned tolerance can produce qualified products at the beginning of a production period, many out-of-specification products will be fabricated once the tooling elements become degraded.

Furthermore, the manufacturing cost of different cases, represented by the summation of reciprocals of all the tolerances (Equation (27)) are compared in Table 8. When degradation is not considered, the tolerances are allocated nonuniformly which results in a manufacturing cost reduction of 20.6%, compared to the uniform 0.25 mm tolerance scheme. When process degradation is considered, product quality is ensured throughout the production without increasing the manufacturing cost from that of the uniform 0.25 mm tolerance scheme. Since defective products will be unavoidably produced under the uniform 0.25 mm tolerance scheme the actual cost is even higher for the empirical method when the quality-loss related costs such as rework, labor, and material waste are counted. Overall, process-oriented tolerance allocation can deliver a high quality product at a comparably lower cost.

## 4. Conclusions

This paper has presented a systematic methodology for *process-oriented tolerancing* in multi-station *manufacturing* processes, with a detailed technical development conducted in the context of multi-station *assembly* processes. The concept of process-oriented tolerancing expands current tolerancing practices that focus on bounding the errors related to product variables, to explicitly include process variables. The resulting methodology expands the concept of "part interchangeability" into "process interchangeability," which is critical in meeting the increasing requirements related to the suppliers selection and benchmarking or outsourcing.

Process-oriented tolerancing includes not only the information on *product* design but also a much broader category of information regarding *process* design and quality requirements. The process-oriented approach integrates design and manufacturing and can thus optimally allocate tolerances to process variables of the whole system with a remarkably low manufacturing cost. Furthermore, the process-oriented approach can integrate stochastic process information (which is usually hard to include in the traditional product-oriented method) in tolerance optimization so that quality satisfaction is ensured for the entire process life-cycle service without raising manufacturing cost. Thus, the shift to the process-oriented paradigm is a critical technological trend as pointed out by Thurow (1992), "In the future sustainable competitive advantage will depend more on new process technologies and less on new product technologies".

## References

- Agrawal, R., Lawless, J.F. and Mackay, R.J. (1999) Analysis of variation transmission in manufacturing processes—part II. *Journal of Quality Technology*, **31**, 143–154.
- Anon (1998) *VSA-3D Release 12.5 User Manual*, Variation System Analysis, Inc., St. Clair Shores, MI 48081.
- Anon (1999) *Optimization Toolbox User's Guide, Version 5*, The MathWorks Inc., Natick, MA 01760.
- Anon (2000) *3-D Variation Simulation*, Dimensional Control Systems, Inc., Troy, MI 48084.
- Archard, J.F. (1953) Contact and rubbing of flat surfaces. *Journal of Applied Physics*, **24**, 981–988.
- Ashgiabor, A., Liu, H.-C. and Nnaji, B.O. (1998) Tolerance control and propagation for the product assembly modeller. *International Journal of Production Research*, **36**, 75–93.
- Bjorke, O. (1989) *Computer Aided Tolerancing*, Tapir Publishers, Trondheim, Norway.
- Cai, W., Hu, S.I. and Yuan, J.X. (1996) Deformable sheet metal fixturing: principles, algorithms, and simulations. *Transactions of the ASME. Journal of Manufacturing Science and Engineering*, **118**, 318–324.
- Camelio, A.J., Hu, S.J. and Ceglarek, D.J. (2001) Modeling variation propagation of multi-station assembly systems with compliant parts. *Transactions of the ASME, Journal of Mechanical Design*, **125**, 673–681.
- Ceglarek, D. and Shi, J. (1995) Dimensional variation reduction for automotive body assembly. *Manufacturing Review*, **8**, 139–154.
- Ceglarek, D. and Shi, J. (1997) Tolerance analysis for sheet metal assembly using a beam-based model. *ASME Design Engineering Division Publication*, **94**, 153–159.
- Ceglarek, D., Shi, J. and Wu, S.M. (1994) A knowledge-based diagnostic approach for the launch of the auto-body assembly process. *Transactions of the ASME. Journal of Engineering for Industry*, **116**, 491–499.
- Chase, K.W. and Parkinson, A.R. (1991) A survey of research in the application of tolerance analysis to the design of mechanical assemblies. *Research in Engineering Design*, **3**, 23–37.
- Choudhuri, S.A. and DeMeter, E.C. (1999) Tolerance analysis of machining fixture locators. *Transactions of the ASME Journal of Manufacturing Science and Engineering*, **121**, 273–281.
- Cunningham, T.W., Matripragada, R., Lee, D.J., Thornton, A.C. and Whitney, D.E. (1996) Definition, analysis, and planning of a flexible assembly process, in *Proceedings of the 1996 Japan/USA Symposium on Flexible Automation*, **2**, pp. 767–778, Boston, MA.

- Ding, Y., Ceglarek, D. and Shi, J. (2000) Modeling and diagnosis of multistage manufacturing process: part I state space model, in *Proceedings of the 2000 Japan-USA Symposium on Flexible Automation*, Ann Arbor, MI.
- Djurdjanovic, D. and Ni, J. (2001) Linear state space modeling of dimensional machining errors. *Transactions of NAMRI/SME*, **XXIX**, 541–548.
- Fainguelernt, D., Weill, R. and Bourdet, P. (1986) Computer aided tolerancing and dimensioning in process planning. *Annals of the CIRP*, **35**(1), 381–386.
- Fratlicelli, B.M.P., Lehtihet, E.A. and Cavalier, T.M. (1999) Tool-wear effect compensation under sequential tolerance control. *International Journal of Production Research*, **37**, 639–651.
- Jeang, A. (1994) Tolerancing design: choosing optimal tolerance specifications in the design of machined parts. *Quality and Reliability Engineering International*, **10**, 27–35.
- Jensen, K.L. and Vardeman, S.B. (1993) Optimal adjustment in the presence of deterministic process drift and random adjustment error. *Technometrics*, **35**, 376–389.
- Jin, J. and Chen, Y. (2001) Quality and reliability information integration for design evaluation of fixture system reliability. *Quality and Reliability Engineering International*, **17**, 355–372.
- Jin, J. and Shi, J. (1999) State space modeling of sheet metal assembly for dimensional control. *Transactions of the ASME Journal of Manufacturing Science & Engineering*, **121**, 756–762.
- Lawless, J.F., Mackay, R.J. and Robinson, J.A. (1999) Analysis of variation transmission in manufacturing processes—part I. *Journal of Quality Technology*, **31**, 131–142.
- Lee, W.-J. and Woo, T.C. (1989) Optimum selection of discrete tolerances. *Transactions of the ASME. Journal of Mechanisms, Transmissions, and Automation in Design*, **111**, 243–251.
- Lee, W.-J. and Woo, T.C. (1990) Tolerances: their analysis and synthesis. *Transactions of the ASME. Journal of Engineering for Industry*, **112**, 113–121.
- Liu, S.C., Hu, S.J. and Woo, T.C. (1996) Tolerance analysis for sheet metal assemblies. *Transactions of the ASME. Journal of Mechanical Design*, **118**, 62–67.
- Mantripragada, R. and Whitney, D.E. (1999) Modeling and controlling variation propagation in mechanical assemblies using state transition models. *IEEE Transactions on Robotics and Automation*, **15**, 124–140.
- Mou, J. (1997) A systematic approach to enhance machine tool accuracy for precision manufacturing. *International Journal of Machine Tools Manufacturing*, **37**, 669–685.
- Ngoi, B.K.A. and Ong, C.T. (1993) A complete tolerance charting system. *International Journal of Production Research*, **31**, 453–469.
- Ngoi, B.K.A. and Ong, C.T. (1998) Product and process dimensioning and tolerancing techniques: a state-of-the-art review. *International Journal of Advanced Manufacturing Technology*, **14**, 910–917.
- Ngoi, B.K.A. and Ong, J.M. (1999) A complete tolerance charting system in assembly. *International Journal of Production Research*, **37**, 2477–2498.
- Quesenberry, C.P. (1988) An SPC approach to compensating a tool-wear process. *Journal of Quality Technology*, **20**, 220–229.
- Rong, Y. and Bai, Y. (1996) Machining accuracy analysis for computer-aided fixture design verification. *Transactions of the ASME. Journal of Manufacturing Science and Engineering*, **118**, 289–299.
- Roy, U., Liu, C.R. and Woo, T.C. (1991) Review of dimensioning and tolerancing: representation and processing. *Computer-Aided Design*, **23**, 466–483.
- Shiu, B., Ceglarek, D. and Shi, J. (1996) Multi-station sheet metal assembly modeling and diagnostics. *NAMRI/SME Transactions*, **XXIV**, 199–204.
- Slocum, A.H. (1992) *Precision Machine Design*, Prentice Hall, NJ.
- Soons, J.A., Theuws, F.C. and Schellekens, P.H. (1992) Modeling the errors of multi-axis machines: a general methodology. *Precision Engineering*, **14**, 5–19.
- Suri, R. and Otto, K. (1999) Variation modeling for a sheet stretch forming manufacturing system. *Annals of the CIRP*, **48**, 397–400.
- Tang, L. C., Goh, C.J. and Lim, S.C. (1988) On the reliability of components subject to sliding wear—a first report. *Sliding Wear*, **22**, 1177–1181.
- Thurow, L. (1992) *Head to Head: the Coming Economic Battle Among Japan, Europe, and America*, Morrow, New York, NY.
- Veitschegger, W.K. and Wu, C. (1986) Robot accuracy analysis based on kinematics. *IEEE Journal of Robotics and Automation*, **2**, 171–179.
- Voelcker, H.B. (1998) The current state of affairs in dimensional tolerancing. *Integrated Manufacturing Systems*, **9**, 205–217.
- Wallbridge, N.C. and Dowson, D. (1987) Distribution of wear rate data and statistical approach to sliding wear theory. *Wear*, **119**, 295–312.
- Whitney, D.E., Gilbert, O.L. and Jastrzebski, M. (1994) Representation of geometric variations using matrix transforms for statistical tolerance analysis in assemblies. *Research in Engineering Design*, **6**, 191–210.
- Wu, Z., ElMaraghy, W.H. and ElMaraghy, H.A. (1988) Evaluation of cost-tolerance algorithms for design tolerance analysis and synthesis. *Manufacturing Review*, **1**, 168–179.
- Zangwill, W.I. (1967) *Nonlinear Programming: A Unified Approach*, Prentice-Hall, Englewood Cliffs, NJ.
- Zhang, G. (1996) Simultaneous tolerancing for design and manufacturing. *International Journal of Production Research*, **34**, 3361–3382.
- Zhang, H. (ed.) (1997) *Advanced Tolerancing Techniques*, J Wiley, New York, NY.

## Biographies

Yu Ding received a B.S degree in Precision Engineering from the University of Science and Technology of China in 1993, a M.S. degree in Precision Instruments from Tsinghua University, China in 1996, a M.S. degree in Mechanical Engineering from the Pennsylvania State University in 1998, and a Ph.D. in Mechanical Engineering from the University of Michigan in 2001. He is currently an Assistant Professor in the Department of Industrial Engineering at Texas A&M University. His research interests are in the area of quality engineering and applied statistics, including process-oriented robust design and tolerancing, in-process variation diagnosis, diagnosability analysis of distributed sensor systems, and optimal sensor system design. His current research is sponsored by the National Science Foundation, Nokia, and the State of Texas Higher Education Coordination Board. He has received a number of awards for his work, including the CAREER Award from the National Science Foundation in 2004 and the Best Paper Award from the ASME Manufacturing Engineering Division in 2000. He is a member of IIE, ASME, SME, and INFORMS.

Jionghua (Judy) Jin received her B.S. and M.S. degrees in Mechanical Engineering, both from Southeast University in 1984 and 1987, and her Ph.D. degree in Industrial Engineering at the University of Michigan in 1999. She is currently an Assistant Professor in the Department of Systems and Industrial Engineering at the University of Arizona. Her research mainly focuses on the fusion of statistical methods with engineering models to develop new methodologies for system modeling, monitoring, fault diagnosis, and control by using applied statistics, wavelets, DOE, reliability, system control, and decision making theory with their applications in various complex systems such as the multistage manufacturing processes of assembly, stamping, semiconductor manufacturing, and transportation service industry. Her current research is being sponsored by the National Science Foundation, SME Education Foundation, US Department of Transportation—Bureau of Transportation Statistics, Arizona State Foundation, and Global Solar Energy Inc. She currently serves on the Editorial Board of the *IIE Transactions on Quality and Reliability*. She is a member of INFORMS, IIE, ASQC, ASME, and SME. She was the recipient of the CAREER Award from

the National Science Foundation in 2002 and the PECASE Award in 2004, and the Best Paper Award from ASME, Manufacturing Engineering Division in 2000.

Dariusz Ceglarek received his Ph.D. in Mechanical Engineering at the University of Michigan in 1994 following his diploma in Production Engineering from Warsaw University of Technology in 1987. He was a research faculty member at the University of Michigan-Ann Arbor from 1995–2000. Currently, he is an Associate Professor in the Department of Industrial Engineering at the University of Wisconsin-Madison. His research interests include the design, control and diagnostics of multistage manufacturing processes; developing statistical methods driven by engineering models to achieve quality improvement; modeling and analysis of product/process key characteristics causality; and reconfigurable/reusable assembly systems. His current research is sponsored by the National Science Foundation, DaimlerChrysler Corp., DCS, and State of Wisconsin's IEDR Program. He was elected as a corresponding member of CIRP and is a member of ASME, SME, NAMRI, IIE, and INFORMS. He has received a number of awards for his work in-

cluding the 2003 CAREER Award from the NSF, the 1998 Dell K. Allen Outstanding Young Manufacturing Engineer Award from the Society of Manufacturing Engineers (SME) and two Best Paper Awards by ASME MED and DED divisions in 2000 and 2001, respectively.

Jianjun (Jan) Shi is a Professor in the Department of Industrial and Operations Engineering at the University of Michigan. He got his B.S. and M.S. degrees in Electrical Engineering at the Beijing Institute of Technology in 1984 and 1987 respectively, and his Ph.D. in Mechanical Engineering at the University of Michigan in 1992. His research interests focus on the fusion of advanced statistical techniques and domain knowledge to develop methodologies for modeling, monitoring, diagnosis, and control for complex manufacturing systems. His research has been funded by the National Science Foundation, the NIST Advanced Technology Program, General Motors, Daimler-Chrysler, Ford, Lockheed-Martin, Honeywell, and various other industrial companies and funding agencies. He is a member of ASME, ASQC, IIE, and SME.

*Contributed by the Reliability Engineering Department*